Title	Secondary School Elementary Mathematics Materials Compilation Series
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Date	28/6/2024

Title	Basic Algebra Notes
Author	Lee Jian Lian
Quality Control	Liu Hui Ling and Lim Wang Sheng

Algebra is basically

- Representing numbers with letters and symbols
- Performing mathematical operations using letters and symbols, together with using numbers.

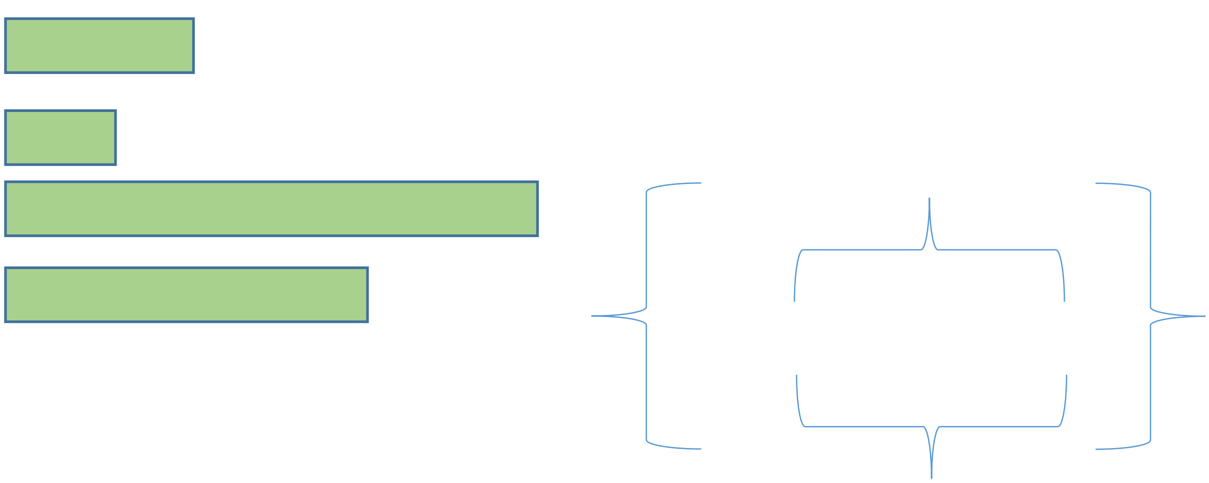
If algebra never existed

- Textbooks will be ridiculously heavy
- Statistical analysis will be impossible to do, as the person writing the data will have virtually endless amount of explanation he/she got to make.
- Certain fields of mathematics, dependent on algebra, cannot exists as well.

Tips to students who are new to algebra:

- Name your variables systematically, in a way you won't get confused
- Don't use letters and symbols that can easily be confused for something else, (I never use Q, O, Z, J, I, S and E, unless the question specify I must use them.)
  [If you go to higher level mathematics, you will understand why E and I are rarely used in mathematical variable representation, they are reserved for specific use.]
- If you ever had to use Z, please strike through the letter in the following manner:
  ~~Z~~
  [To avoid confusion with the number "2"]
- Use visualization to connect model drawings with algebra representation.

For convenience in typing later: I have decided to create a few bars here so they can be copied with a few clicks and on demand. (I don't need to draw again and again, which is tedious even on a computer.) (Teachers with issue with using the computer or word processor can just copy my template and use them.)



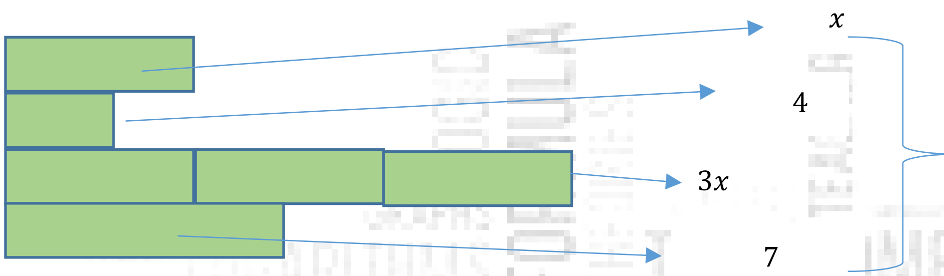
Basic Operands in Algebra:	
Expression	English Meaning
$x + 5$	Add 5 to x
$x - y$	Subtract y from x
$6n = n + n + n + n + n + n$	Multiply n by 6
$\frac{m}{n}$	Divide m by n

Exponents and Roots	
Expression	English Meaning
$n^7 = n \times n \times n \times n \times n \times n \times n$	n to be multiplied by itself 7 times.
$n^{-5} = \frac{1}{n^5} = \frac{1}{n \times n \times n \times n \times n}$	1 divided by total value of n multiplied 5 times in a row.

Example 1:
Solve the following Expression
$x + 7 + 3x + 4$

$= x + 3x + 7 + 4$
$= 4x + 11$

Let's convert it to models and see how they corelate:



Sum up all the values represented by the models and you should get the same expression.

How algebra is translated into models in this case:

1. There are four boxes of the variable x which is equal to $4x$.
2. There are one box of 7 and one box of 4, hence the total value is $4x + 11$.

The above also, in a way, demonstrate the concept of “like” and “unlike” terms.
In any algebraic addition and subtraction, only “like terms” can be added or subtracted in this way.

$2x + 3x = 5x$
$5x - 4x = x$

Arithmetic demonstration of concept:
$2x + 3x = 5x$

Since $2x = 2 \times x$,
We can pretend the value of x is 5.

In this case:
$2x = 2 \times 5$ and $3x = 3 \times 5$
$10 + 15 = 25$

The other way is true as well and valid:
$2x + 3x = 5x = 5 \times 5 = 25$

More Complicated Subtraction and Addition and Demonstration by Purely Algebra Approach and How to Check Your Answers.

Example 2:

$3x + 5 - 3x + 4$

$= 3x - 3x + 5 + 4$ (Group "Like Terms" together)

$= 0 + 9 = 9$ (Final Answer)

To verify your answer is correct, what you need to do.

Pretend $x = 2$

$(3 \times 2) + 5 - (3 \times 2) + 4$

$= 6 + 5 - 6 + 4$

$11 - 6 + 4 = 5 + 4 = 9$

Example 3:

$15xy - 5 + 2 - 13xy$

$= 15xy - 13xy - 5 + 2$ (Group "Like Terms" Together)

$= 2xy - 5 + 2$

$= 2xy - 3$ (Final Answer)

To verify the correctness of your answer, you need to

Pretend $x = 2$ and $y = 3$ in your original question.

$(15 \times 2 \times 3) - 5 + 2 - (13 \times 2 \times 3)$

$(90) - 5 + 2 - (78)$

$= 90 - 5 + 2 - 78 = 9$

Then pretend $x = 2$ and $y = 3$ in your answer.

$(2 \times 2 \times 3) - 3$

$= 12 - 3$

$= 9$

Some teachers also call this type of answer checking as "Checking Answers by Value Substitution" as you are indeed "Substituting the Value" of the letters by another value.

Problem Solving (Addition and Subtraction of Algebra):

To solve a problem sum using algebra, while it varies slightly in different questions, we observed the following steps are necessary.

1. Find out what are the unknowns, needed to solve the question.
2. Define the known and unknown variable properly using algebra letters and numbers.
3. Construct the equation or expression needed to solve the question.
4. Solve the equation and find the value of unknown variable.

The easiest example I've seen on books that can demonstrate this:

Example 4:

The sum of three consecutive numbers is 84. What is the smallest of the three numbers.

1	Find out the unknown.	The smallest number out of the three consecutive numbers.
2	Define the unknown and known properly	In this case, the unknown value (smallest number) is defined as x. Since they said the numbers are in consecutive order, we can define the subsequent two numbers as $x + 1$ and $x + 2$
3	Construct an Equation	Sum means, add the three numbers together, so we have: $x + (x + 1) + (x + 2) = 84$
4	Solve the Equation and find out what is the unknown value, in this case, it is x.	$3x + 1 + 2 = 84$ $3x + 3 = 84$ $3x = 84 - 3$ $3x = 81$ $x = 27$ [Final Answer] Thus, the smallest number is 27.

Performing Algebra Addition and Subtraction (Using Brackets) Demonstration and Concept:

The below two example seems extremely straightforward:
$a + b = a + b$
$a - b = a - b$

It is the below few example students often get confused about and are careless enough to lose a total of 7 marks in exams.

$a + (-b) = a - b$
$a - (-b) = a + b$

$(-a) + b = b - a$
$(-a) + (-b) = (-b) - a$

Performing Algebra Multiplication and Division (Using Brackets) Demonstration and Concept:

The below two examples are straightforward.
$a(b) = ab$
$a \div b = \frac{a}{b}$

Once again students can get really confused when negative signs are involved.
$a(-b) = -ab$
$-a(b) = -ab$
$(-a)(-b) = ab$

$a \div (-b) = -\frac{a}{b}$

$(-a) \div b = -\frac{a}{b}$

$(-a) \div (-b) = \frac{a}{b}$

Example 5

Simplify $-2(3x-4+6x)$

We must take note of 2 things in the above example, firstly, the bracket needs to be removed, secondly, that the expression in the bracket has to be multiplied by a negative value outside the bracket.

$-2(3x)-2(-4)-2(6x)$	Remove Brackets by multiplying values outside of the brackets with the value inside the bracket.
$=-6x+8-12x$	Group "Like Terms" Together
$=\ -18x+8$	Final Answer

Example 6

Simplify $\frac{1}{3x-1}+\frac{3x}{5}$

This time, we will be performing calculations and simplifying algebra fractions. Students may begin to think how to start, and I would like you all to recall how you add fractions in your primary school by converting the denominator to their least common multiple.

Evaluate $\frac{2}{5}+\frac{1}{4}$

$$\frac{2}{5}+\frac{1}{4}=\frac{2}{5}\left(\frac{4}{4}\right)+\frac{1}{4}\left(\frac{5}{5}\right)=\frac{8}{20}+\frac{5}{20}=\frac{13}{20}$$

In the case of the example $\frac{1}{3x-1}+\frac{3x}{5}$, it turns out the lowest common multiple of the denominator is $5(3x-1)=15x-5$

$$\frac{1}{3x-1}+\frac{3x}{5}$$

After converting the denominator to their lowest common multiple, we will also multiply the numerator according to what led us to the new denominator values.

$$=\frac{1}{(3x-1)}\left(\frac{5}{5}\right)+\frac{3x}{5}\left(\frac{3x-1}{3x-1}\right)=\frac{5(1)}{5(3x-1)}+\frac{3x(3x-1)}{5(3x-1)}$$

$$=\frac{5}{15x-5}+\frac{9x^2-3x}{15x-5}=\frac{5}{15x-5}+\frac{(9x^2-3x)}{15x-5}$$

$$=\frac{9x^2-3x+5}{15x-5}$$

Example 7
(Dealing with Negative Signs in Algebraic Fractions)

Simplify $\frac{2x}{9}-\frac{2x-5}{7}$

After converting the denominator values to their lowest common multiple, we will also multiply the numerator according to what led us to the new dominator values, which get us to the following expression

$$\frac{2x}{9}\left(\frac{7}{7}\right)-\frac{(2x-5)}{7}\left(\frac{9}{9}\right)$$

$$=\frac{14x}{63}-\frac{9(2x-5)}{63}$$

$$=\frac{14x-9(2x-5)}{63}$$

$$\frac{14x-18x+45}{63}$$

$$=\frac{-4x+45}{63}$$

- Beware of Negative Signs
- All fractions should be combined into a single denominator right after the denominator has a Lowest Common Multiple. (Prevent Careless Mistakes)
- Be careful when dealing with brackets.

Example 8:
(Dealing with a type of algebraic expression where you have to deal with both whole number coefficients and algebra fractions in the same expression)

First, let me explain what I mean by dealing with both whole number coefficients and fraction within one a single row.

It refers to an algebraic expression that resembles the following expressions.

$$\frac{a}{b} + nc$$

Where n is the coefficient of c and n is a whole number

To make it even clearer, let me demonstrate a proper question below.

Simplify $\frac{5x+6}{5} - 4x$

In this case, we just need to multiply the $-4x$ by 5.
The above expression is then transformed into $\frac{5x+6}{5} - \frac{5(4x)}{5(1)}$

Rationale:
For the sake of putting both the expression within the same denominator, we need to create a denominator for $4x$.

Since $4x = \frac{4x}{1}$, we can convert the value to $\frac{4x}{1}$.

And because the left side of the expression shows $\frac{5x+6}{5}$, we need to cross multiply the right ride of the expression by 5 as well, giving us a value of $\frac{5(4x)}{5} = \frac{20x}{5}$.

This means, the expression is now being written with a common denominator and can be rewritten as:
$\frac{5x+6-20x}{5}$

Which can be simplified into:
$$\frac{-15x+6}{5}$$

Title	Basic Algebra – Changing Subject of Formula [Final Version]
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [Part of NYP Mentoring Club]
Date	13/3/2018

Realizing in my class, a lot of students have issues with algebra and I anticipate similar problems to occur with its slightly trickier counterpart, formula manipulation. I decided to write some materials for my own students to use. Due to involvement in public projects, I decided to publish this file through various people, hoping it will help other students as well.

Definitions

Changing Subject of Formula – Making an algebraic symbol in a formula (anything, from letters, symbols) appear as a standalone variable on the left-hand side of the formula. [Will be described in greater detail and will explain how my definition works]

Notation used in this document	
Notation	**Meaning**
RHS	Right-Hand Side of a formula
LHS	Left-Hand Side of a formula

Important things to take note (Or be cautious about)

1. Squares and Square Roots (Negative and Positive Answers are possible in some cases, will be demonstrated in the subsequent pages)
2. (Rarely) If they specify the subject of formula should be a variable that only has a positive value (Example include, distance and speed), be aware that the negative value should be rejected in this case as distances and speed cannot have any negative signs.

Explaining Point 1 in Greater Detail Using Arithmetic:

Supposed we have the number "4" and we want to square root the number.

$\sqrt{4} = 2 \; or - 2$

Both 2 and (-2) satisfy the requirement of being a square root value of 4,

$2^2 = 4$ AND $(-2)^2 = 4$

Example 1:
Make c the subject of the formula.
$E = mc^2$

When they ask you to make c the subject of the formula, they are asking you to rearrange the formula in a way that c must appear as a single algebraic letter on the LHS of the formula.

Steps Taken	Explanation
$\frac{E}{m} = c^2$	Divide both sides by m to isolate c^2
$c^2 = \frac{E}{m}$	Change c^2 to the LHS of the formula
$c = \pm\sqrt{\frac{E}{m}}$	Square rooting c^2 on the LHS gives two possible values, positive and negative, on the right-hand side of the formula.

Example 2:
Make $\boldsymbol{m}$ the subject of the formula
$E = \frac{1}{2}mv^2$

Steps	Explanation
$2(E) = mv^2$	Get rid of fraction by multiplying both sides by 2.
$mv^2 = 2E$	Change sides of the formula
$m = \frac{2E}{v^2}$	Divide both sides by v^2 to isolate and get m

Example 3

Make the b the subject of the formula (Not taken from physics, self-created)

$\frac{3}{f}+\frac{1}{bc}=\frac{1}{g}$

Steps	Explanation
$\frac{1}{bc}=\frac{1}{g}-\frac{3}{f}$	Isolate $\frac{1}{bc}$ from formula
$\frac{1}{bc}=\frac{1(f)-3(g)}{fg}$	Create a common denominator for RHS
$\frac{1}{bc}=\frac{f-3g}{fg}$	
$bc=\frac{fg}{f-3g}$	When a fraction gets inversed on the LHS, the same applies to the RHS. Can be explained in the following manner: $\left(\frac{1}{bc}\right)^{-1}=\left(\frac{f-3g}{fg}\right)^{-1}$ $bc=\frac{fg}{f-3g}$
$b=\frac{(fg)}{c(f-3g)}$	Divide both sides by c to isolate b on the LHS

Example 4:

Make v the subject of the formula.

$$t = \left(\frac{s}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

Steps taken	**Explanation**
$t^2 = \frac{s^2}{1-\left(\frac{v^2}{c^2}\right)}$	Removing any square root signs in the formula by squaring both sides of the formula
$t^2\left(1-\frac{v^2}{c^2}\right) = s^2$	Cross Multiply
$t^2 - \frac{tv^2}{c^2} = s^2$	Removal of brackets
$-\frac{tv^2}{c^2} = s^2 - t^2$	Minus "t^2" from both sides
$-tv^2 = c^2(s^2 - t^2)$	Cross Multiply
$v^2 = \frac{c^2(s^2 - t^2)}{-t}$	Divide both sides by $-t$ to isolate and get v^2
$v = \pm\sqrt{\frac{c^2(s^2 - t^2)}{-t}}$	v is the subject of the formula, square rooting the LHS gives two possible values on the RHS, positive and negative.

Example 5:

Given the following formula,

$$t = \frac{n}{\sqrt{1-\frac{2GM}{rc^2}}}$$

Make c the subject of the formula.

Steps taken	Explanation
$t^2 = \frac{n^2}{1 - \frac{2Gm}{rc^2}}$	Remove Square Root
$t^2\left(1 - \frac{2Gm}{rc^2}\right) = n^2$	Cross Multiply
$t^2 - \frac{2t^2Gm}{rc^2} = n^2$	Remove Brackets
$-\frac{2t^2Gm}{rc^2} = n^2 - t^2$	Minus t^2 from both sides
$-2t^2Gm = rc^2(n^2 - t^2)$	Cross Multiply
$rc^2 = \frac{-2t^2Gm}{(n^2 - t^2)}$	Divide both sides by $(n^2 - t^2)$
$c^2 = \frac{-2t^2Gm}{r(n^2 - t^2)}$	Divide both sides by r to isolate and get c^2
$c = \pm\sqrt{\frac{-2t^2Gm}{r(n^2 - t^2)}}$	Square root the value on both sides, once again, square rooting the LHS gives two possible value on the RHS.

Title	Basic Concept of Inequality in Mathematics (Secondary 1E/1NA/2NT)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [NYP Mentoring Club]
Notes	This is designed for use by students from every stream (NA/E/NT)

Refer to This if You are confused!

Symbols Used	Meaning
$>$	Greater Than
$<$	Less Than

Applications of inequalities ranges from

- Science (Physics, Engineering and Chemistry, to explain that certain science or engineering related situation will only occur at a given range of values)
- Computing and Game Development (To code computer game and program logic)

Follow the following rules when solving inequalities	
If you add, subtract, multiply or divide on one side of the inequality by any values, do the same on the other side. If you add, subtract any types of values (positive or negative) on both sides, the signs are unchanged. If you multiply or divide positive values on both sides, the inequality signs are unchanged.	Example 1: $m+5>10$ $m+5-5>10-5$ Example 2: $m-5>10$ $m-5+5>10+5$ $m>15$ Example 3: $6m>15$ $\frac{6m}{6}>\frac{15}{6}$ $m>\frac{15}{6}$ $m>\frac{5}{2}$
If you multiply or divide negative values on both sides, the inequality signs must change.	Example 4: $-m>5$ $(-1)(-m)<(-1)(5)$ $m<5$

	Example 5: $-10m > 5$ $-\frac{10m}{-10} < \frac{5}{-10}$ $m < -0.5$

Further Proof of Concept Given Below Using Numbers	
Adding of Positive Numbers	$5 > 2$ $5 + 4 > 2 + 4$ $9 > 6$ 9 is still greater than 6
Subtracting of Positive Values	$10 > 2$ $10 - 2 > 2 - 2$ $8 > 0$ 8 is still greater than 0
Multiplication of Positive Values	$16 > 1$ $16 \times 2 > 1 \times 2$ $32 > 2$ 32 is still greater than 2
Division of Positive Values	$12 > 9$ $\frac{12}{3} > \frac{9}{3}$ $4 > 3$ 4 is still greater than 3
Adding of Negative Numbers	$-2 < 5$ $-2 + (-5) < 5 + (-5)$ $(-2) - 5 < 5 - 5$ $-7 < 0$ -7 is still less than 0
Subtracting of Negative Values	$-18 < 6$ $-18 - (-4) < 6 - (-4)$ $-18 + 4 < 6 + 4$ $-14 < 2$ -14 is still less than 2

*Multiplication of Negative Values	$-15 > -22$ $-1(-15) < -1(-22)$ $15 < 22$ After multiplying -1 to the inequality, 15 is less than 22
*Division of Negative Values	$-30 < 1$ $\frac{-30}{-5} > \frac{1}{-5}$ $\frac{30}{5} > 0.2$ $6 > 0.2$ After dividing -5 to the inequality, 6 is greater than 0.2

Introducing "Greater Than or Equals to" and "Less Than or Equals to"

Refer to this if you are confused

Symbols Used	Meaning
$\geq$	Greater Than or Equals To
$\leq$	Less Than or Equals to

The way the "Greater than or Equals to" and "Less Than or Equals to" work is similar to examples I wrote in the last few pages.

Just a few things I hope students take note of

(Common sense, but well, some students are that careless)

- If the question didn't mention symbols "$\leq$" OR "$\geq$", you don't go and write those symbols down, using incorrect symbols results in loss of marks.

If question mentions symbols "$\leq$" OR "$\geq$", similar rules applies

- If you add or subtract any numbers from the inequality, the symbols are unchanged.
- If you multiply or divide positive numbers from the inequality, the symbols are unchanged
- If you multiply or divide negative numbers from the inequality,
  $\geq$ changes to $\leq$
  $\leq$ changes to $\geq$

Example 1	Solve the following inequality $x + 6 \geq 12$ $x + 6 - 6 \geq 12 - 6$ $x \geq 6$
Example 2	Solve the following inequality $y - 6 \leq 42$ $y - 6 + 6 \leq 42 + 6$ $y \leq 48$
Example 3	Solve the following inequality $\frac{m}{2} \geq 5$ $2\left(\frac{m}{2}\right) \geq 2(5)$ $m \geq 10$
Example 4	Solve the following inequality $7m \geq 14$ $\frac{7m}{7} \geq \frac{14}{7}$ $m \geq 2$

Example 5***	Solve the following inequality $-\frac{m}{5} \leq 15$ $-5\left(-\frac{m}{5}\right) \geq -5(15)$ $m \geq -75$ ***Note how the sign changes.
Example 6***	Solve the following inequality $-7m \geq 28$ $\frac{-7m}{-7} \leq \frac{28}{-7}$ $m \leq -4$ ***Note how the sign changes.

Title	Mathematics (Lowest Common Multiple and Highest Common Factor)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [Mentoring Club, Nanyang Polytechnic] (Editor) Lee Jian Lian
Date	24/2/2018

Understanding some schools chose to teach the law of indices in Secondary 3 rather than Secondary 1, I will like to explain various ways on how the laws of indices works before proceeding to introduce the concepts as it will be frequently used in the documents.

Notation	English	Explanation
a^b Example: $3^2 = 3 \times 3$ $2^3 = 2 \times 2 \times 2$	a to the power of b	It means to multiply a by itself b number of times. a is the base while b is the power or the exponent.

Law of Indices Relevant to this topic

$(a^b)(a^c) = a^{b+c}$

$$\frac{(a^b)}{(a^c)} = a^{b-c}$$

$(a^b)^c = a^{bc}$

In General

$\sqrt[c]{a^b} = a^{\frac{b}{c}}$

Specifically

$\sqrt{a} = a^{\frac{1}{2}}$

$\sqrt[3]{a} = a^{\frac{1}{3}}$

Definitions:

A prime number is a number greater than 1, that can only be divisible by itself or 1.

Recalling how you find Lowest Common Multiple and Highest Common Factor in Primary School.

Example 1:
Find the Lowest Common Multiple and Highest Common Factor of 18 and 15.

Highest Common Factor
(Listing down all factors and drawing conclusion of the highest factor shared between both values.)

Numbers	Arithmetic Form	Listed
18	1×18 2×9 6×3	1,2,3,6,9,18
15	1×15 3×5	1,3,5,15

The highest common factor is deduced to be 3.

Lowest Common Multiple
(Listing down every multiple possible for both values and to draw conclusion of the lowest multiple shared between both values)

Numbers	Listed	
18	18,36,54,72,90,108	
15	15,30,45,60,75,90,105	

The lowest common multiple is therefore 90.

The problem with this method, it works only for small numbers, as the numbers get larger and larger, listing the factors and multiples becomes increasingly difficult and tiring and thus we need a more efficient method to find lowest common multiples and highest common factors.

Example 2:
Find the highest common factor of 168 and 560.

Step 1: Divide both numbers repeatedly by prime numbers until you get 1 and thus unable to divide further.

Divisor	Dividend	Quotient
2	560	280
2	280	140
2	140	70
2	70	35
5	35	7
7	7	1

Divisor	Dividend	Quotient
2	168	84
2	84	42
2	42	21
7	21	3
3	3	1

Step 2: Express Numbers in Index Notation.
$560 = 2^4 \times 5 \times 7$
$168 = 2^3 \times 3 \times 7$

Step 3: Highlight values with common bases.
$560 = 2^4 \times 5 \times 7$
$168 = 2^3 \times 3 \times 7$

Step 4: Take value which satisfy two conditions: Highlighted (Common Base), Lowest Power of the two rows.

What we get is	2^3 and 7

Step 5:
Multiply the values to get Highest Common Factor

HCF = $2^3 \times 7 = 56$

Example 3:
Find the lowest common multiple of 45 and 378.

Step 1: Divide the values repeatedly by prime numbers until you get 1, thus unable to divide any further.

Divisor	Dividends	Quotients
3	45	15
3	15	5
5	5	1

Divisor	Dividends	Quotients
2	378	189
3	189	63
3	63	21
7	21	3
3	3	1

Step 2: Express Numbers in Index Notation.
$45 = 3^2 \times 5$
$378 = 2 \times 3^3 \times 7$

Step 3: Highlight values that satisfy the following condition.
If base common in both rows, pick the one with highest power.
If the base value is only found on either row, highlight the value as well.

What we get is	$2, 5, 3^3, 7$

Step 4: Multiply the highlighted values together to get Lowest Common Multiple.

LCM = $2 \times 3^3 \times 5 \times 7$ =1890

Example 4:

(a) Express 2700 as a product of its prime factors.
(b) Given that 2700h is a perfect square, write down the smallest possible value of h
(c) Given that 2700k is a perfect cube, write down the smallest possible value of k.

4(a)

Divisor	Dividends	Quotient
2	2700	1350
2	1350	675
5	675	135
5	135	27
3	27	9
3	9	3
3	3	1

Product of Prime Factors (Index Notation): $2^2 \times 5^2 \times 3^3 = 2700$

4(b)
To solve this question, I would like to introduce you to the vocabulary needed to solve the question and the rules of solving similar questions.

For a number to be even called a perfect square, it needs to:

- Produce an integer when square rooted.

For 2700h to be a perfect square, it needs to satisfy the following requirements: Multiplying h to 2700 should be able to ensure all the power of the prime factors of 2700h are divisible by 2.

Which bring us to the law of indices

$a^b \times a^c = a^{b+c}$

Apparently, since the only base value where the power isn't divisible by 2 is 3^3
And that $(3^3)(3) = 3^4$

$2700h = 2^2 \times 5^2 \times 3^3 \times 3 = 2700(3)$
$h = 3$ (Final Answer)

4(c)

For a number to be called a perfect cube, it needs to

- Produce an integer when being cube rooted.

For 2700k to be a perfect cube in this case, multiplying k to 2700 results in:

- Powers of prime factors of 2700k should all be divisible by 3.

As you see from the following values, there are two values for which the powers are not divisible by 3.

$2^2 \times 5^2 \times 3^3 = 2700$

Since $2^2 \times 2 = 2^3$
And $5^2 \times 5 = 5^3$

$2700k = 2(2^2) \times 5(5^2) \times 3^3 = 2^3 \times 5^3 \times 3^3$

$k = 2 \times 5 = 10$ (Final Answer)

(Questions all obtained from CASCO Mathematics 4B Assessment Book)

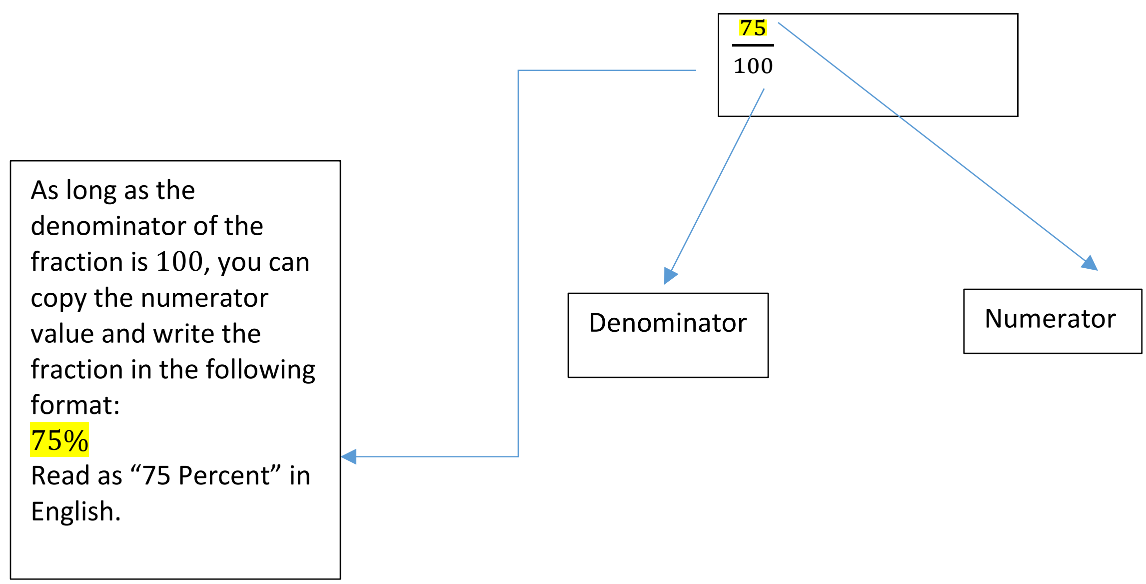
Title	Percentages – Basics and Applications (Secondary 1)
Editor	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club] [This document is created through a joint collaboration with Liu Hui Ling, she wrote the original version several years ago, found a copy below her desk, sent to me a scanned copy and ask me to improve it if necessary.]
Date	21/9/2018

Concepts to Cover Include the Following

- Percentage Basics
- Practical Applications – Basic Applications
- Practical Applications – Percentage Change
- Practical Application – Reverse Percentages

I want to start off by explaining what is required for this topic as introduction, the basic concepts you need to know are the following:

- Percentages are numerator values within a fraction, for which the denominator is widely understood to have a value of 100.



Basically, from my above illustration, I showed that percentages are just another form of fraction, just that the denominator is 100 and the way to express this fraction is replaced by the "%" symbol.
From this, we will be able to further on understand other concepts and application.

In Example 1, we have the following lesson objectives in mind.

- Finding value of a percentage rate of a base value
- Finding total value, of which a certain percentage gets added to the base value

Basic Applications
Example 1. [Applying Percentage to Get Final Value]

Product ID	QT-531A
Name	Potato Chips (1 Can)
Pricing (Before GST)	$7.20
GST Rate	8.5%
GST Tax (In $)	(a)
Pricing (After GST)	(b)

Fill in the above blanks.
To do the above question, we need to understand the goal of the question itself, the question asks us to do the following

- Find GST Tax Rate – (Deriving value of a percentage, given base value)
- Find Pricing After GST – (Deriving final target value, given percentage.)

(a)
Formula for deriving value of percentage of a base value goes as follows

$$Percent\ Rate \times \ Base\ Value = Value\ of\ Percentage$$

In this case, the GST Tax is the "Value of Percentage", while the percent rate is 8.5%
Which can be written as the following statement
GST Tax in $ as follows:
$8.5\% \times \$7.20$ = $0.612
$\$0.612 = \$0.61\ (Nearest\ Cents)$

(b)
Formula for deriving final value, when certain percentage of a base value gets added in.

$$(100\% + Percent\ Rate)(Base\ Value) = Final\ Value.$$

Pricing of Potato Chips after GST as Follows
$(100\% + 8.5\%) \times \$7.20 = 108.5\% \times \$7.20 = \$7.812$
$\$7.812 = \$7.81\ (Nearest\ Cents)$

After obtaining the answer, the answers are to be filled in as follows:

Product ID	QT-531A
Name	Potato Chips (2 Bag)
Pricing (Before GST)	$7.20
GST Rate	8.5%
GST Tax (In $)	$0.61
Pricing (After GST)	$7.81

In Example 2, the following concepts are to be illustrated

- Finding value of percentages of base value.
- Finding final values, of which a certain percentage gets subtracted from the base value.

Example 2.

A staff member of ABC Company keys in the following information into the inventory system, and accidentally left some blanks in the process. You will have to fill in the missing information and provide clear workings for your answer.

Product ID	QN-242C
Name	ABC Electronic Dictionary Model QN-242C [Exam Approved]
Original Pricing	$25.00
Discounts	Yes
Discount Rate	20%
Discount Amount (In $)	(a)
Discounted Price	(b)

(a)

Formula for deriving value of percentage of a base value goes as follows

$$Percent\ Rate \times\ Base\ Value = Value\ of\ Percentage$$

In this case, the percent rate refers to the discount rate, while the original value refers to the original pricing.

Discount Amount

$20\% \times \$25.00 = \$5.00$

(b)

Formula for deriving final value of which a certain percentage gets removed from the base value

$$(100\% - Percent\ Rate)(Base\ Value) = Final\ Value$$

Pricing of ABC Electronic Dictionary QN-242C after discount applied

$(100\% - 20\%) \times \$25 = \$20.00$

I filled in the blanks as follows.

Product ID	QN-242C
Name	ABC Electronic Dictionary Model QN-242C [Exam Approved]
Original Pricing	\$25.00
Discounts	Yes
Discount Rate	20%
Discount Amount (In \$)	\$5.00
Discounted Price	\$20.00

In Example 3, it will be different, the goal of Example 3, is to find the target percentage, given the value of the target and base value. (Calculator Guide at Last Page of This Topic)

Example 3.

In a country XYZ election, a group of 200 people is interviewed. Out of the group of people interviewed, about 137 people are supportive of candidate A, about 51 people are supportive of candidate B, the remaining being unsure.

(a) Find the percentages of the number of people within the group who are supportive of candidate B.

3(a)

$$Desired\ Percentage = \left(\frac{Value}{Base\ \ Value}\right) \times 100\%$$

In this case,

Target Value is the number of people in support of candidate B.

Base Value is the total number of people interviewed.

Target Percentage is the percentage of people in support of candidate B.

Percentage of people in support of candidate B as follows:

$$\left(\frac{51}{200}\right) \times 100\% = 25.5\%$$

Change in Percentage
Example 4
The following data shows the stock market situation of XYZ Indices Index.

Current Point	1298
Change (Refers to the amount of value increased or decreased compared to the previous trading day, as denoted by +/- symbols respectively.)	+31
52 Week High	1330
52 Week Low	1069

Answer the following questions
(a) What is the percentage increase or decrease of XYZ Indices Index from the previous trading day to the current points?
(b) Calculate the percentage increase from the 52-Week-Low value to the current number of points.
(c) Calculate the percentage decrease from the 52-Week-High value to the current number of points.

Formula for Percentage Increase and Decrease as Follows

$$Percentage\ Increase = \frac{Later\ Value - Previous\ Value}{Previous\ Value} \times 100\%$$

$$Percentage\ Decrease = \frac{Previous\ Value - Later\ Value}{Previous\ Value} \times 100\%$$

4(a)

Since $Later\ Value - Previous\ Value = Difference\ in\ Both\ Values$,
the difference between the previous day stock index points can be rewritten as follows

$$Percentage\ Increase = \frac{Difference\ in\ Both\ Values}{Previous\ Value} \times 100\%$$

$Difference\ In\ Both\ Values = 31$
$Previous\ Value = 1298 - 31 = 1267$

$$Percentage\ Increase = \frac{31}{1267} \times 100\% = 2.4467\%$$

$Percentage\ Increase = 2.45\%\ (3\ significant\ figures)$

4(b)

Previous Value = 52 Week Low = 1069
Later Value = 1298

$$Percentage\ Increase = \frac{Later\ Value - Previous\ Value}{Previous\ Value} \times 100\%$$

$Percentage\ Increase = \frac{1298-1069}{1069} \times 100\% = 21.4\%\ (3\ Significant\ Figures)$

4(c)

$Previous\ Value = 52\ Week\ High = 1330$
Later Value = 1298

$$Percentage\ Decrease = \frac{Previous\ Value - Later\ Value}{Previous\ Value} \times 100\%$$

$Percentage\ Increase\ \frac{1330-1298}{1298} \times 100\% = 2.47\%\ (3\ Significant\ Figures)$

Reverse Percentages
If you are being given the final value, percentages and asked to find the base value, the usual approach to doing question simply don't work here, instead, you rely on the following methods.

5(a)
John buys a pair of shoes from a local fashion store. At a discounted rate of 15%, the pair of shoes cost $17.50. Find the amount of money she has to pay for the pair of shoes if the discount wasn't offered.

$$Final\ Value = 17.50$$

$$\frac{100\%}{(100 - 15)\%} \times \$17.50 = \$20.59$$

Cost of Shoes (Before Discount) = $\$20.59$ (Nearest Cents)

5(b)
ABC Company reported earnings of $\$257\ 036$ after paying taxes in 2005. Given the tax rate for his company is 7.35%. Compute the amount of earning before the company paid its taxes on that year.

$$Final\ Value = 257\ 036$$

$\frac{100\%}{(100-7.35)\%} \times 257\ 036 = \$277426.88$ (Nearest Cents)

Reminder (Calculator Matters)

The way you write your workings and answers isn't necessarily the same as how you input into a calculator, as illustrated in the below example.

Written Format	Calculator Input
Objective: Find the percentage equivalent of a fraction. $\frac{23}{46} \times 100\% = 50\%$	Objective: Find the number of % equivalent of a fraction. $\frac{23}{46} \times 100 = 50$ This value is the number of percent equivalent of the fraction value $\frac{23}{46}$.

This is because calculators interpret the following statements literally in the following way.

$\frac{23}{46} \times 100\%$ is interpreted as $\frac{23}{46} \times \frac{100}{100} = \frac{23}{46} \times 1 = \frac{23}{46}$

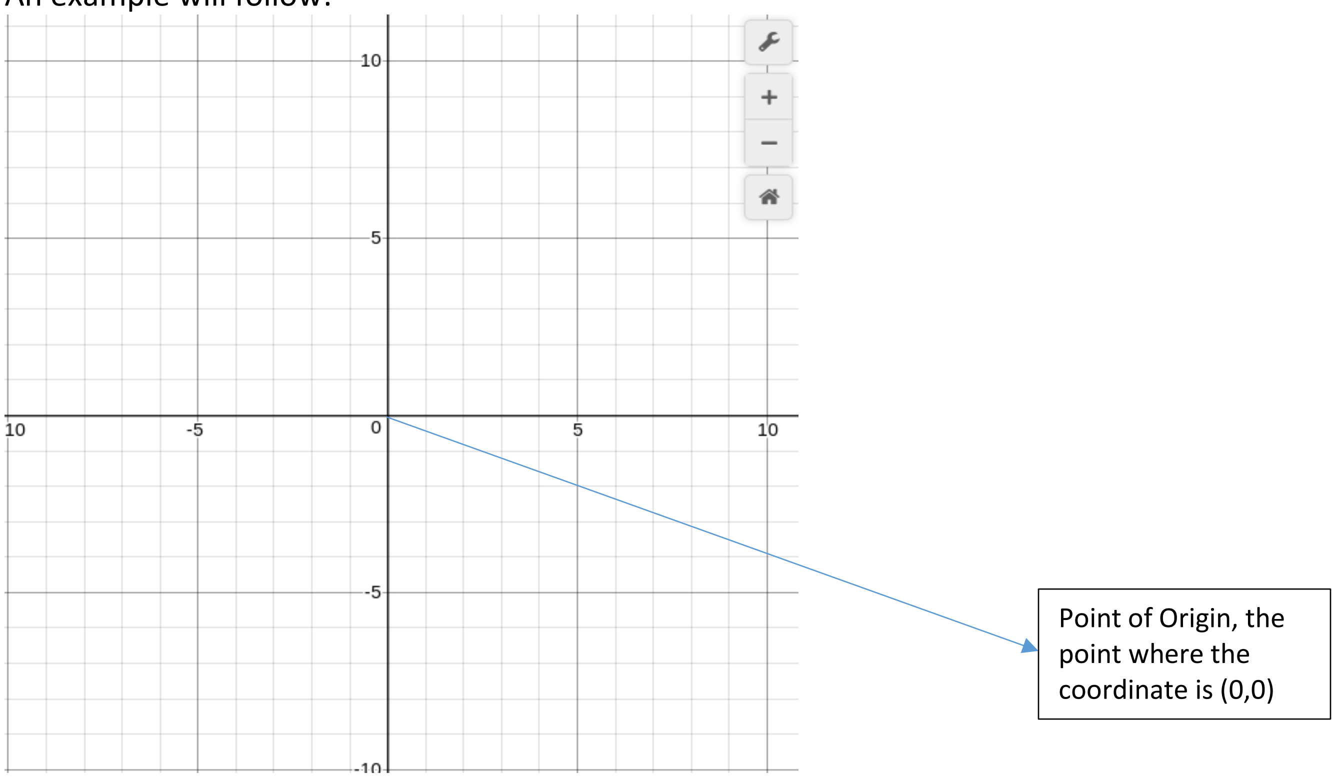
I want to remind students that when keying in to the calculator, remember not to key in "× 100%" **if the purpose is to find the percentage equivalent on a fraction**, instead you key the fraction, multiplied by 100 and convert the value from fraction to decimal, if necessary, and the decimal value displayed on the screen is the number of %. You write the decimal value displayed on the screen followed by the percent symbol (%).

Title	Coordinate Geometry – Basic Overview
Date	18/1/2019
Author	Lee Jian Lian Liu Hui Ling, Ngee Ann Polytechnic

Coordinate Geometry appears in many ways in our lives, the idea of Coordinate Geometry in simple terms is to perform Mathematical calculations and applications of calculations on a plane, in this case, we focus on the cartesian coordinate system.

Basic principle surrounding the idea of cartesian coordinate system.
You have a horizontal axis called the x-axis, a vertical axis called the y-axis. Both axis intersect each other at $90°$ (Also called perpendicular to each other).

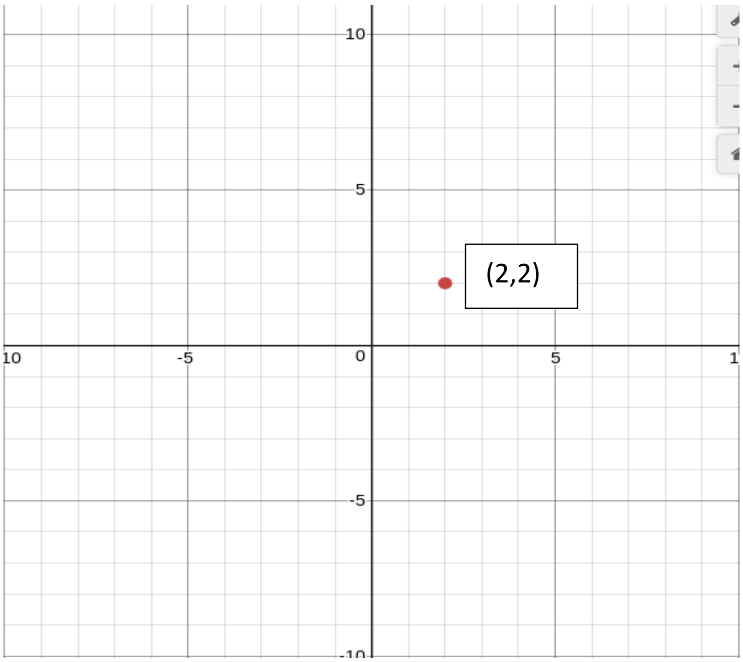
An example will follow:



I want to remind students to mark out x-axis and y-axis during exam or doing homework, the computer software used doesn't do that.

In a Cartesian Coordinate system, we can things like points, lines and curves.

Points



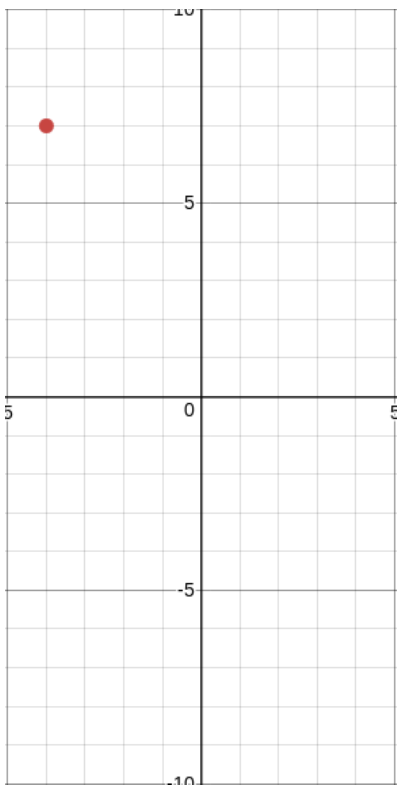
Points are typically expressed in the following notation as follows

$(x - coordinate, y - coordinate)$

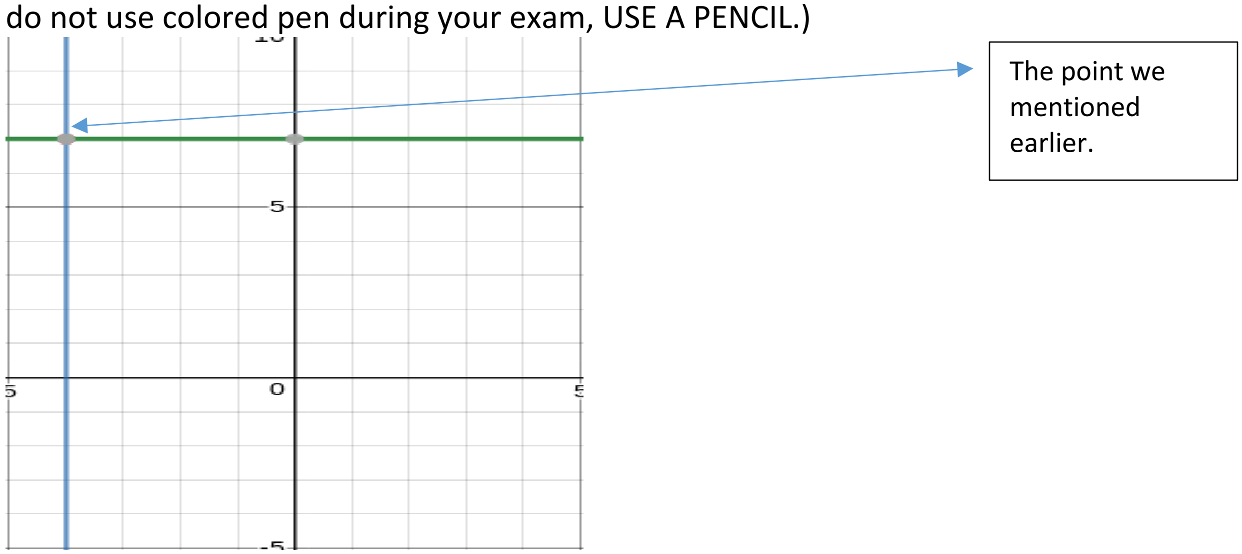
We can do another example to illustrate this point.

Showing how to find a coordinate of a point

Step 1. Mark out the point of the coordinate mentioned in the question or obtained from previous parts of the question.



Step 2. From the X axis, use a ruler or gently use a pencil to trace the shortest path from the X-axis to the point. From the Y-Axis, use a ruler or gently use a pencil to trace the shortest path from the Y-Axis to the point in question. (I use color to indicate clearly but do not use colored pen during your exam, USE A PENCIL.)

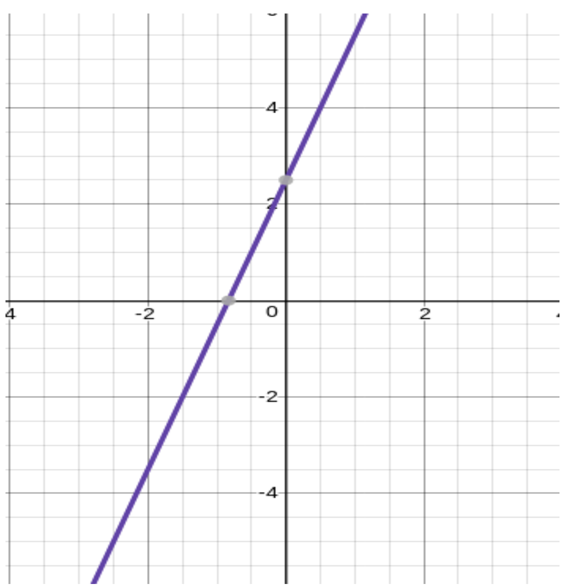


Step 3.
Deduce the Coordinate Values for X and Y.
x —coordinate refers to the value for which the path passes through both the point and the $x - axis$, and y —coordinate refers to the value for which the path passes through both the point and the y —axis.

Scaling
Each big box, as noted with darker black lines, represents a 5-unit by 5-unit grid space. Each small box as noted with lighter black lines, represents a 1-unit by 1-unit grid space.

Therefore, from the number of units away from the y —axis, the point 4 units to the left of the point of origin and 7 units above the point of origin. Giving us a coordinate value of $(-4,7)$

Lines
Apart from Points, we can also have lines and curves on a Cartesian Coordinate system.
Lines

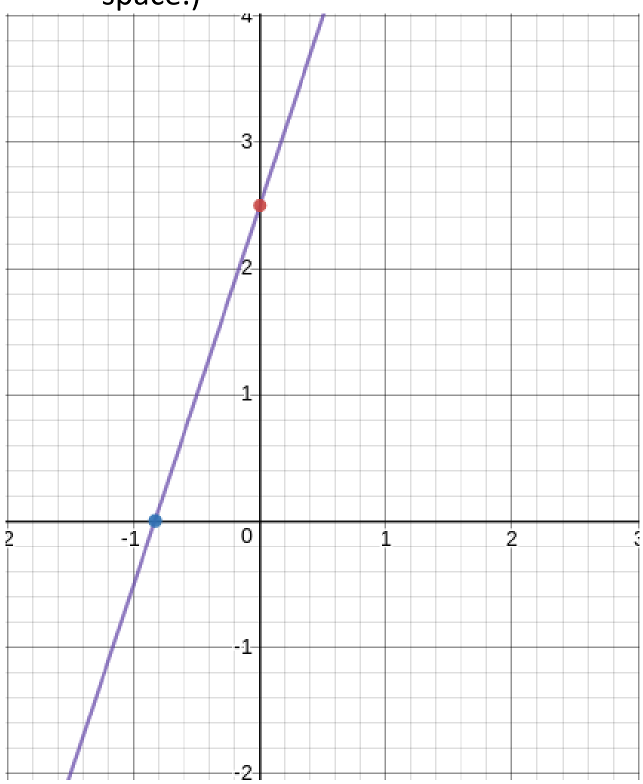


Two things we are particularly interested in

- Slope (Steepness) of the Line (Also called the gradient)
- Point where the Line crosses the Y-axis (Also called the Y-Intercept)

To find Gradient, follow the following steps

1. Select a point on the graph, called the coordinate of the point (x_1, y_1) and select another different point on the graph, calling the coordinate of the point (x_2, y_2) [Note the graph scaling is different this time, each small box represents a 0.2 unit by 0.2-unit grid space and each darker box, represents a 1 unit by 1 unit of grid space.)



(x_1, y_1)	$(-0.85, 0)$
(x_2, y_2)	$(0, 2.5)$

Apply gradient formula

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 2.5}{(-0.85) - 0} = 2.941$$

$y - intercept$ can be approximated by the same methods established earlier and we can tell from the graph the y —intercept is 2.5 units above the point of origin, therefore the y-intercept is 2.5.

(When point fall between 2 tiny square, select the mid-point of the tiny square as your coordinate value.)

[I also would like to say, just try your best in obtaining the best approximation possible.]

Title	Mathematics Coordinate Geometry (Lower Secondary)
Editor	Lee Jian Lian
Date	1/4/2018

Formula List Required

General Form (or General Equation) of a Straight Line:

$y = mx + c$

Where m is the gradient and c is the y-intercept (Or the value of y when $x = 0$)

[If line is vertical, the gradient is undefined.]

Given Coordinates (x_1, y_1) AND (x_2, y_2) are coordinates on a straight line.

Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

Length of Line (In Units) = $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

Basic Steps Briefing

To find the equation of a line, you need to do the following steps:

1. Write down the coordinates of points where the line will pass through
2. Calculate gradient by using the above mentioned gradient formula
3. Substitute coordinate into the general equation form of a straight line to find out the y-intercept.
4. Collect Gradient Information and Y-Intercept to deduce equation
5. (If applicable) Make use of deduced equation to solve any remaining questions

Notes:

If gradient is undefined but the question insist that you should write an equation for the line, you can write the equation of the line in terms of x. (e.g. $x = 5$)

[Questions all Taken from CASCO 4B Assessment Book]

Example 1

The coordinates of A and B are $(3,-5)$ and $(-1,-9)$ respectively.

Find

(a) the Length of AB.

(b) the equation of Line AB.

(c) the value of p if the point $C(2p,6)$ lies on the line AB

(d) the equation of the line l which has the same gradient as the line $3x+y=1$ and passing through point C.

<table>
<tr><th>Parts</th><th>Formula and Reasoning</th><th>Steps Taken</th></tr>
<tr><td>(a)</td><td>Length of Line (In Units) =
$\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$</td><td>
<table>
<tr><th>y_2</th><th>y_1</th><th>x_2</th><th>x_1</th></tr>
<tr><td>-9</td><td>-5</td><td>-1</td><td>3</td></tr>
</table>
Value of $y_2-y_1=(-9)-(-5)=-4$

Value of $x_2-x_1=(-1)-3=-4$

$\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$ =
$\sqrt{(-4)^2+(-4)^2} = \sqrt{32}=5.6569=$
$5.66\ units\ (3sf)$
</td></tr>
<tr><td>(b)</td><td>Gradient = $\frac{(y_2-y_1)}{(x_2-x_1)}$</td><td>
<table>
<tr><th>y_2</th><th>y_1</th><th>x_2</th><th>x_1</th></tr>
<tr><td>-9</td><td>-5</td><td>-1</td><td>3</td></tr>
</table>
Value of $y_2-y_1=(-9)-(-5)=-4$
Value of $x_2-x_1=(-1)-3=-4$

Therefore gradient, $m=1$

$y=mx+c$
</td></tr>
</table>

	Substitute associated coordinates into the graph equation to find y-intercept.	You can see how it resembles and correspond to the coordinates in the form of: $y_2 = m(x_2) + c$ $(-9) = 1(-1) + c$ $(-9) - 1 = c$ $c = -8$ Thus, equation of the Line AB is: $y = x - 8$
(c)	Substitute associated coordinate into the equation to find the value of x, when $x = 2p$ and $y = 6$	The equation is once again, in the form: $y = x - 8$ $6 = x - 8$ $6 + 8 = x$ $2p = x = 14$ $p = 7$
(d)	Line L has the same gradient as $3x + y = 1$ At point $C(14,6)$	Rearrange in the form of $y = mx + c$ $y = -3x + 1$ Gradient of Line $L = -3$ $y = mx + c$ $6 = -3(14) + c$ $c = 48$ Thus, Equation of Line L is $y = -3x + 48$

Title	Mathematics (Direct & Inverse Proportion)
Date	22/3/2018
Author	Lee Jian Lian

This material is designed to teach students basics of inverse and direct proportion.

The following table describes the notation I used when teaching my friend's siblings about this topic, you can consider using my notation or consult your teachers about what notation to use.

Notation	English Name	Meaning
$(Subscript)_{Given}$	"Subscript Given Value"	Values derived from given information in the question.
$(Subscript)_{New}$	"Subscript New Value"	Values used to solve later parts of the question.

Please also state which part of the question you are trying to solve in the question paper before writing down how you solve the question. I will also demonstrate how I use those notation, you will realize how clear my working is. Students with problems with producing clear workings can consider a similar way of writing down workings like me.

Direct Proportion

Two values are said to be in direct proportion when the following conditions are met:

- Each time when the value of the former is increased, the latter will increase as well.
- Both values can be expressed in the following form $y = bx$, where b is a constant.

(Some teachers and questions may use different letters but they essentially mean the same thing.)

Example 1:
y is directly proportional to x. Given that $y = 144$ and $x = 12$. Find the value of y when $x = 7$.

$y_{given} = 144$
$x_{given} = 12$
Since y is directly proportional to x,
$y_{given} = b(x_{given})$
$144 = b(12)$
$b = 12$

Since $x_{new} = 7$
$y_{new} = b(x_{new})$
$y_{new} = 12(7)$
$y_{new} = 84$

When $x = 7, y = 84$

Inverse Proportion

Two values are said to be in inverse proportion when the following conditions are met:

- Each time when you increase the former value, the latter value decreases. (And each time when decrease the former value, the latter value increases.)
- Both values can be expressed in the form of $y = \frac{b}{x}$

Example 2:

L is inversely proportional to the $\sqrt{M}$. When $M = 100, L = 35$.
Find the value of M, when $L = 700$.

$$L_{given} = 35$$
$$M_{given} = 100$$
$$\sqrt{M}_{given} = \sqrt{100} = 10$$

$$L_{given} = \frac{b}{\sqrt{M}_{given}}$$

$$35 = \frac{b}{\sqrt{100}}$$

$$b = 35(10) = 350$$

Since $L_{new} = 700$

$$700 = \frac{350}{\sqrt{M}_{new}}$$
$$700\sqrt{M}_{new} = 350$$
$$\sqrt{M}_{new} = \frac{350}{700} = \frac{1}{2}$$
$$M_{new} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Title	Mathematics (Maps and Scales)
Date	20/3/2018
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic Lee Jian Lian (Editor)

Logic and terms that you need to understand before proceeding:

Why study this topic?
This topic, not surprisingly, has many practical applications. That's precisely how people in the past all the way till now, create maps and allow users of the map to understand, how far should you go from one place to another when you are only just given a map.

Even in the modern days, when smartphones map applications are gradually taking over, that's exactly how smartphones can tell you how long you are supposed to walk on a street to get to another place. The map application is set to interpret a certain "zoom" level as a certain scale on a map, zooming in the map reduces ratio between literal map distance and actual distance, zooming out the map does the opposite.

Terms used in this topic	**Meaning**
D_{map}	The literal distance of any two mentioned points within a map.
D_{actual}	The actual distance of any two mentioned points within a map, in real life.
A_{map}	The literal area of any shape as displayed on a map.
A_{actual}	The actual area of any mentioned shape within a map, in real life.

Formula List

Scale of Map Formula (In representative fraction)	Scale = $\frac{D_{map}}{D_{actual}}$ (Where D_{map} $\textit{has to be reduced to 1 to be expressed as a fraction}$.
Ratio of areas between map area and actual area.	$\left(\frac{A_{map}}{A_{actual}}\right) = \left(\frac{D_{map}}{D_{actual}}\right)^2$

Questions All Taken from CASCO Mathematics 4B Assessment

Example 1:

On a map, a length of 4 cm represents an actual distance of 1 km.

Calculate

a) the actual distance, in kilometers, represented by 26 cm on the map,
b) the scale of the map in the form of $1{:}\,n$,
c) the area on the map, in square centimeters, which represent an actual area of $9\ km^2$

To solve this question, we need to derive the following information

Details	**Steps Taken**
Ratio between map distance and actual distance in real life	Scale = $\frac{D_{map}}{D_{actual}}$ = $\frac{4cm}{1km} = \frac{1\ cm}{0.25\ km}$ We must make sure the units are the same to establish the scale's representative fraction: $\frac{1\ cm}{250\ m} = \frac{1\ cm}{25000\ cm}$ Remove the units to get Scale = $\frac{1}{25000}$
Ratio between map area and actual area in real life	Area Scale (Without Unit) = $\left(\frac{A_{map}}{A_{actual}}\right) = \left(\frac{D_{map}}{D_{actual}}\right)^2 = \left(\frac{1}{25000}\right)^2$ Area Scale (With Unit) = $\left(\frac{1\ cm}{0.25\ km}\right)^2 = \frac{1\ cm^2}{0.0625\ km^2}$

a)

Let y be the actual distance in real life.

Given Scale Fraction = $\frac{1\ cm}{0.25\ km}$ = $\frac{26\ cm}{y\ km}$

$y = 26 \times 0.25 = 6.5$

Thus, distance in real life = 6.5 km

b)

Given Scale Fraction = $\frac{1}{25000}$

When written in ratio form it is $1{:}\,25000$

c)

Given Area Scale Fraction (With Unit) = $\frac{1\ cm^2}{0.0625\ km^2}$

Let x be the area on map

$$\frac{x\ cm^2}{9\ km^2} = \frac{1\ cm^2}{0.0625\ km^2}$$

$$x = \frac{9}{0.0625} = 144$$

Thus, area on map = 144 cm^2

Title	Mathematics (Solving Quadratic Equations)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	20/3/2018

Mathematics Terms
"Root", "Solution" and "Answers" means the same thing in this topic. They are a set of numbers that can satisfy a specified equation.
Quadratic Equation – an equation for which the highest power within the equation is 2.
LHS – Left-hand Side
RHS – Right-hand Side
"Find Solution" and "Solving" mean the same thing in this topic.

Identifying a quadratic equation

Based on the above definition of a quadratic equation, a quadratic equation is any equation that is, in the following form or can be re-arranged in the following form:

$$ax^2 + bx + c = 0$$

Where $a \neq 0$

[If $a = 0$, the equation effectively reduces to $bx + c = 0$, which is linear, not quadratic.]

Three approaches used to solve quadratic equation I am going to teach

- Factorization/Cross Method (Calculator Tips and Tricks Provided as well)
- Quadratic Formula
- Completing the Square Method

Factorization/Cross Method

Algebraic Identity Approach

Only work when the given equation resembles any 1 of the 3 algebraic identity provided as below.

- $a^2 + 2ab + b^2 = 0$
- $a^2 - 2ab + b^2 = 0$
- $a^2 - b^2 = 0$

As all the expression on the LHS (Left-Hand side) of the equation can be factored into the following:

- $a^2 + 2ab + b^2 = (a + b)^2$
- $a^2 - 2ab + b^2 = (a - b)^2$
- $a^2 - b^2 = (a + b)(a - b)$

Basic Demonstration

Example 1

Solve $x^2 + 6x + 9 = 0$

Noting how the above equation resembles the expression:

$$a^2 + 2ab + b^2 = (a + b)^2$$

As $2ab = 2(x)(3) = 6x$

Factorize the LHS of the equation into
$x^2 + 6x + 9 = (x + 3)^2$
$(x + 3)^2 = 0$

$(x + 3)(x + 3) = 0$
$x + 3 = 0 \; OR \; x + 3 = 0$
Final Answer: $x = -3$

Example 2:
Solve $x^2 - 8x + 12 = -4$

In this case, the RHS of the equation isn't zero, so, we have to rearrange the equation in the following way:
$x^2 - 8x + 12 + 4 = -4 + 4$
$x^2 - 8x + 16 = 0$

We can tell the above expression on the LHS of the equation resembles the following:

$$a^2 - 2ab + b^2 = (a - b)^2$$

As $(-2ab) = -2(x)(4) = -8x$

Factor the LHS of the equation into the following:
$x^2 - 8x + 16 = (x - 4)^2$
$(x - 4)(x - 4) = 0$
$x = 4\ OR\ x = 4$

Final Answer: $x = 4$

Example 3:
Solve $x^2 - 64 = 0$

We can tell the above expression on the LHS of the equation resembles the following:

$$a^2 - b^2 = (a + b)(a - b)$$

$a^2 = x^2$
$-b^2 = -64$

Thus, the expression can be factored in the following:
$x^2 - 64 = (x + 8)(x - 8)$

$(x + 8)(x - 8) = 0$
$x + 8 = 0\ OR\ x - 8 = 0$
Final Answer: $x = -8\ OR\ x = 8$

Cross Method (Calculator Tricks)
Look out for any options on your calculator that can help you to solve any quadratic equations by keying in values $a, b\ and\ c$ as you will need to use it.

In the case of FX-95SG-PLUS
You press "Mode", "3" "3" to have access to such functionality, other calculators may have similar functionality.

Troubleshooting and Error Messages (Using FX-95 SG-PLUS calculator as example)
If you get a "Math Error" message, check the following:
Did you put $a = 0$? [a cannot be 0]
Did you key in value of a wrongly as something else which is not number?

If you get "Syntax Error" message, check the following:
Make sure you didn't accidentally type in any symbols apart from numbers into the calculator in such mode.

Example 4:
Solve $x^2 - 7x - 330 = 0$
[Due to technical difficulties, it is very difficult to draw and demonstrate cross method on computer. I investigated in methods to do so but none of them produces a nice, decently looking drawing. I want to remind students here, if you are using calculator to solve quadratic equation directly, you must also demonstrate how it can be done using cross method, quadratic formula or completing the square on your question paper or answering booklet. Failure to include the method of doing results in loss of marks.]

After keying in the value of $a = 1, b = -7\ and\ c = -330$ into the calculator
I get the following values as shown:

Final Answer
$x = -15\ OR\ x = 22$

Quadratic Formula Approach

As the name suggest, quadratic formula is a formula based method used to solve quadratic equation, it is (sort of) derived from completing the squares which I will share with you in the subsequent pages.

In any quadratic equation that you are trying to solve, there can be 2 real and distinct roots, equal roots or no real roots, this would depend on the discriminant $b^2 - 4ac$ (which I will not further discuss here as discriminant related problems appear more in Additional Mathematics question papers, as compared to Elementary Mathematics.)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 5

Solve $6x^2 + 7x - 15 = 0$

In this case, you are going to substitute the quadratic formula with coefficient values shown in the equation.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad OR\ x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 + \sqrt{7^2 - 4(6)(-15)}}{2(6)} \quad OR\ x = \frac{-7 - \sqrt{7^2 - 4(6)(-15)}}{2(6)}$$

$$x = \frac{-7}{12} + \frac{\sqrt{409}}{12} \quad OR \quad x = -\frac{7}{12} - \frac{\sqrt{409}}{12}$$

$$x = 1.10\ OR\ x = -2.27\ (3sf)$$

Example 6

Solve $x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 + \sqrt{1^2 - 4(1)(1)}}{2(1)} \quad OR\ x = \frac{-1 - \sqrt{1^2 - 4(1)(1)}}{2(1)}$$

Situation (On FX-95 SG Plus)	What to expect when such an attempt to solve the equation is made.
You solve equation manually (i.e. key in one by one).	"Math Error" Message.
You solve it using the quadratic equation solver functionality within the calculator	Numbers that comes in the form of $a + bi$ In this case, I get the following values: $x = -0.5 + 0.866i\ OR\ x = -0.5 - 0.866i$
**Different calculator operates differently.	

Respond to both answers by writing the following on your answering booklet

Final Answer: No Real Roots.

Completing the squares approach

Example 7

Solve $x^2 + 7x + 9 = 0$

Rearrange equation into the following format:

$(x \pm h)^2 \pm k = 0$

Which means you have to do the following steps:

Add $\left(\frac{c}{2}\right)^2$ between bx and c and Subtract $\left(\frac{c}{2}\right)^2$ after c, which can be shown as follows:

$$x^2 + 7x + \left(\frac{7}{2}\right)^2 + 9 - \left(\frac{7}{2}\right)^2 = 0$$

$$\left(x + \frac{7}{2}\right)^2 - 3.25 = 0$$

$$\left(x + \frac{7}{2}\right)^2 = 3.25$$

$$\left(x + \frac{7}{2}\right) = \pm\sqrt{3.25}$$

$$x + 3.5 = \sqrt{3.25} \quad OR\ x + 3.5 = -\sqrt{3.25}$$

$$x = -\sqrt{3.25} - 3.5\ OR\ x = \sqrt{3.25} - 3.5$$

$$x = -5.30\ OR\ x = -1.70\ (3sf)$$

Example 8

Special case of Completing the Square where $a > 1$ or $a < -1$

Solve $7x^2 - 22x + 10 = 0$

We need to first, divide LHS of the equation by a.

$$\frac{7x^2 - 22x + 10}{7} = 0$$

$$x^2 - \frac{22}{7}x + \frac{10}{7} = 0$$

In this case, we have to add $\left(\frac{c}{2a}\right)^2$ between $\frac{b}{a}$ and $\frac{c}{a}$ and subtract $\left(\frac{c}{2a}\right)^2$ after $\frac{c}{a}$.

$$x^2 - \frac{22}{7} + \left(\frac{\frac{22}{7}}{2}\right)^2 + \frac{10}{7} - \left(\frac{\frac{22}{7}}{2}\right)^2 = 0$$

$$x^2 - \frac{22}{7} + \left(\frac{22}{14}\right)^2 + \frac{10}{7} - \left(\frac{22}{14}\right)^2 = 0$$

$$\left(x - \frac{22}{14}\right)^2 - \frac{51}{49} = 0$$

$$x - \frac{22}{14} = \pm\sqrt{\frac{51}{49}}$$

$$x = \sqrt{\frac{51}{49}} + \frac{22}{14}\ OR\ x = -\sqrt{\frac{51}{49}} + \frac{22}{14}$$

Final Answer:

$x = 2.59\ or\ x = 0.551\ (3sf)$

Title	Coordinate Geometry – Quadratic Graph
Author	Guta Chorgen *(1998 ~ 2016)*
Date	Circa 2013 18/1/2019 [Reconstructed]

Properties of Quadratic Functions as Plotted on a Graph Paper.

- Minimum or Maximum point is exactly on the same point where the graph is symmetrical

Because of this property, methods for finding line of symmetry of a quadratic function will also work for finding the minimum or maximum point.

$For\ a > 0$ in $ax^2 + bx + c = 0$, the graph has a minimum point.
$For\ a < 0$ in $ax^2 + bx + c = 0$, the graph has a maximum point.

To find the line of symmetry or the minimum or maximum point of a quadratic function, we use the following formula.

$$x_{symmetry} = -\frac{b}{2a}$$

Question 1.
Find the Coordinate of Minimum/Maximum Point of $y = -5x^2 - 6x + 3$
Since $a < 0$, the Quadratic Function has a maximum point.

$$x_{symmetry} = -\frac{-6}{2(-5)} = -0.6$$

Substituting $x_{symmetry}$ into $-5x^2 - 6x + 3$
We get the following value as y
$-5(-0.6)^2 - 6(-0.6) + 3 = -2.4$

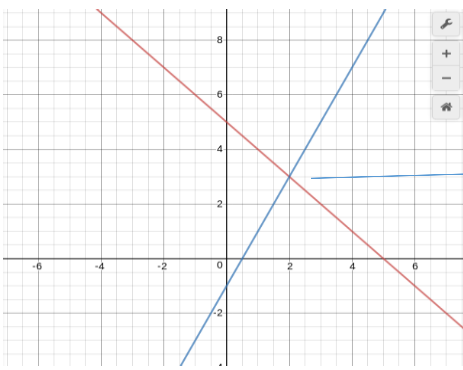
Therefore, maximum point is $(-0.6, 4.8)$

Title	Simultaneous Equations (Secondary 2)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	25/12/2018

The purpose of this article is to provide an overview to simultaneous equation and to teach the methods required to solve such system of equations.
The purpose of a simultaneous equation is to find the point of intersection (also called the coordinate of the intersection) of 2 different lines.

Graphical Illustration as follows
The below graph, created with the help of an App named "Desmos", illustrates the following simultaneous equation

$$x + y = 5$$
$$2x - y = 1$$



The coordinates of Intersection Point (The x and y coordinate) are the so-called "solutions" of a simultaneous equation.

Setting aside the graphical method, where students are asked to convert simultaneous equations to $y = mx + c$ form, graph them literally on a graphing paper or graphing grid space to find the coordinate(s) of intersection point(s), which most of the time provides a reasonably good approximation, there are ways to obtain exact solutions using algebraic methods, namely

- Elimination Approach
- Substitution Approach

I will be providing students with some idea how to judge which method is better in certain situation if the question does not require a specific method to be used.
If the question requires you to use a specific method, <u>you must use that method, or marks may be lost.</u>

Elimination Approach
Question 1
Solve the simultaneous equations:

$2x - 5y = -1$	Equation 1
$6x - 4y = 30$	Equation 2

Why I think elimination method is more suitable for this question.

- Done in fewer steps (Just multiply by their common factors)
- The question creates a situation that is more favorable to using elimination method.

From Equation 1 to Equation 1A
$3(2x - 5y) = 3(-1)$
$6x - 15y = -3$

Equation 2 - Equation 1A
$(6x - 4y) - (6x - 15y) = 30 - (-3)$
$6x - 4y - 6x + 15y = 33$
$-4y + 15y = 33$
$11y = 33$

Equation 3
$y = 3$

Substitute Equation 3 into Equation 1
$2x - 5(3) = -1$
$2x = -1 + 15$
$2x = 14$
$x = 7$

Answer: $y = 3, x = 7$

Substitution Approach
Question 2
Solve the following simultaneous equations.

$y = 2x - 5$	Equation 1
$7x - 4y - 12 = 0$	Equation 2

Why I think substitution method is more suitable in this case

- It takes less effort to substitute the values as you just need to substitute equation 1 into equation 2.

Substitute Equation 1 into Equation 2.
$7x - 4(2x - 5) - 12 = 0$
$7x - 8x + 20 - 12 = 0$
$-x + 8 = 0$
$-x = -8$

Equation 3
$x = 8$

Substitute Equation 3 into Equation 1.
$y = 2(8) - 5$
$y = 16 - 5 = 11$

$y = 11$

$x = 8, y = 11$

Calculator Guide for Simultaneous Equations (Casio FX-95 SG PLUS and FX-96 SG PLUS Family)

Continuing from our guide on solving simultaneous equations, there is going to be a guide for using calculator to check if solutions to your simultaneous equation is correct as well.
For FX-95 SG PLUS and FX-96 SG PLUS family of calculator, the steps goes as follows

Step 1. Rearrange your simultaneous linear equation in the following format, as the calculator mentioned will only be able to recognize the following order as your input.

$ax + by = c$
$dx + fy = g$

Example as illustrated
$2x + y = 5$
$x - y = 6$

Step 2. Press the following buttons and please do it in a correct order
[Mode], [3], [1]

Step 3
Under the first row of the table displayed, you key in the each of the following value, pressing the [=] button after each value is entered, replacing the letters with values you obtained in step 1.

$$a, b, c$$

If we use the example as our input, you should be able to see the following on your screen

a	b	c
2	1	5

Step 4
After keying in the value of c, and pressing [=] button, your cursor will automatically jump to the next row, you key in the following value, pressing the equal sign after each value is entered, replacing the letters with the value you obtained from step 1.

$$d, f, g$$

If we use the example as our input, you should be able to see the following on your screen

a	b	c
2	1	5
1	-1	6

Step 5: Press [=] button after you also filled in the second row and the solution to the simultaneous equation should appear.

Step 6: Now, you should see the value of x displayed on the screen

Step 7: To find the value of y, press the down arrow located just below the calculator screen.

(Note: I only used Casio Family of Scientific Calculators, I wrote this documentation to highlight to students the fact that most calculator used in the education industry can solve such equation. I encourage students or users of other brands of calculator to research for how solving equations can be done for other brands.)

(You still must show your workings even though your calculator can solve simultaneous equation. That's why, I said in the previous page, this calculator functionality is only used for checking if your answers are correct.)

Title	Basic Statistics – Mean, Median, Mode and Standard Deviation Application on Ungrouped Data
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	13/9/2018

For Ungrouped Data	
Mean	Meaning: The value of the total of all observations divided by the number of observations. $\bar{X} = \frac{\Sigma x}{N}$ $\bar{X}$ refers to the mean value of the observation in question Σx refers to the total of all observations N refers to the number of observations [I assume you all understand how to calculate mean, but if you still need help in understanding, proceed to the next few pages, I have condensed the example for standard deviation and mean together since calculating mean is necessary to calculate standard deviation.]
Mode	Meaning: The most frequently occurring value in the observation. Example: 2,6,0,5,5,5 The mode in the above example is 5, since "5" appear most frequently.

Median	Meaning: The middle value of observation. **Example 2 (If the number of observation is odd.)** Given 9,5,3,1,2 [There are only 5 observations in the data given, the number of observation is odd] Rearrange in ascending order (if the values are indeed not in ascending order) 1,2,3,5,9 Median Position Number = $\left[\frac{Number\ of\ Observations+1}{2}\right]$ Median Value = The value represented by the median position number. In this case the median position number is calculated by $(\frac{5+1}{2}) = 3$ The third value in ascending order is the median value, in this case, the value is 3. **Example 3 (If the number of observation is even)** Given 2,4,8,10,12,15 (Already in ascending order, so we do not need to rearrange) Median to be calculated as follows Position number of 2 median numbers to be computed as follows $First\ Middle\ Position\ Number = \left(\frac{Number\ of\ Observations}{2}\right)$ $Second\ Middle\ Position\ Number = \left[\frac{(Number\ of\ Observation)}{2} + 1\right]$ In this case, the 2 middle-position number are represented by the following:

	First Middle Position Number $\frac{6}{2} = 3$ and the 3rd value in ascending order is 8 Second Middle Position Number $\frac{6}{2} + 1 = 4$ and the 4th value in ascending order is 10 Add up the 2 middle-position-number and divide by 2 Median = $\frac{8+10}{2} = 9$
Standard Deviation	Refers to how spread out is the data. (At the present level, that's is all you need to know, at higher level, there are other ways to measure how spread out the data is which will not be covered here.) $Standard\ Deivation = \sqrt{\frac{\Sigma(x-\bar{X})^2}{N}}$ **Example 4** Compute the standard deviation of the following given data. 5,0,9,7,2 The mean of the data $(\bar{X})$ in this case is $\frac{5+0+9+7+2}{5} = \frac{23}{5} = 4.6$ Standard Deviation to be computed as follows. $\Sigma(x-\bar{X})^2 = (5-4.6)^2 + (0-4.6)^2 + (9-4.6)^2$ $+(7-4.6)^2 + (2-4.6)^2 = 53.2$ $N = 5$ $\sqrt{\frac{\Sigma(x-\bar{X})^2}{N}} = \sqrt{\frac{53.2}{5}} = 3.26\ (3sf)$

Title	Statistics – Mean and Standard Deviation of Grouped Data
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	25/12/2018

In the previous article, I demonstrated how to deal with ungrouped data, this article extends the idea towards dealing with finding grouped data.

Firstly, I need to explain and illustrate the differences between grouped and ungrouped data.

- Ungrouped Data – Refers to data that has not been processed or divided into different groups.
- Grouped Data – Refers to data that has been processed and divided into groups. The group typically refers to intervals of values corresponding to a specific number of subjects (also called the frequency).

An illustration will be provided below demonstrating the differences and how one can convert an ungrouped data into grouped.

Example (Ungrouped Data)
Height of Students in Class 721 of XYZ Tuition Center (in Centimeters)
91.4, 131.0, 102.0, 132.0, 140.0, 97.2, 142.7, 100.0, 143.0, 129.0, 155.0, 137.0, 98.4,

If presented in a grouped data form, it looks like this

Height Class Intervals	Frequency
$90 < x \leq 100$	4
$100 < x \leq 110$	1
$120 < x \leq 130$	1
$130 < x \leq 140$	3
$140 < x \leq 150$	3
$150 < x \leq 160$	1

When you are being asked to convert ungrouped data to grouped data, be extremely careful with the inequality sign as misreading the inequality sign can result in inaccurate or incorrect answers as well as misinterpretation of the table.
(Note the difference between $>, <$ and $\geq, \leq$)

The formula for mean of grouped data as follows

$$\bar{X} = \frac{\Sigma f(x)}{\Sigma f}$$

$\bar{X}$	The mean value $\frac{\Sigma f(x)}{\Sigma f}$
$\Sigma f(x)$	Literally implies the following: You literally multiply the mid-point of the class interval by its corresponding frequency value. Repeat the process for every row and add the results obtained from performing the operation on every row to get $\Sigma f(x)$
Σf	Literally implies the following You obtain the frequency value from every row and sum them up to get Σf.

The formula for standard deviation of grouped data as follows:

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma f(x^2)}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

**Write Standard Deviation in Full. Do not use symbols like σ, s *and etc*.

$\frac{\Sigma f(x^2)}{\Sigma f}$	Literally means, you square the mid-point of the class interval and multiply the result with the frequency, repeat the process for every row and add those up together to get $\Sigma f(x^2)$ Then you divide $\Sigma f(x^2)$ by the sum of frequency from all rows to get $\frac{\Sigma f(x^2)}{\Sigma f}$
$\left(\frac{\Sigma fx}{\Sigma f}\right)^2$	I have already introduced you all how to find the mean value, this literally means, you square the mean value to get $\left(\frac{\Sigma fx}{\Sigma f}\right)^2$

Sometimes you may hear.
"Mean of Squares – Square of Mean", then square root the result to get the standard deviation. This is a good way to remember.

Example 1.
Find the mean and standard deviation of the following data, the following data is part of a research on how well students in a tuition class score in their test.

Marks	Number of Students
$50 < x \leq 60$	15
$60 < x \leq 70$	28
$70 < x \leq 80$	18
$80 < x \leq 90$	13
$90 < x \leq 100$	8

For me, I prefer writing the mid-point value of each class interval beside the table. But if you want to completely guarantee you won't get confused, you can either use a pen different color from the question paper or simply draw out the table.

Marks (Mid-Point Value in Parentheses)	Number of Students
$50 < x \leq 60$ (55)	15
$60 < x \leq 70$ (65)	28
$70 < x \leq 80$ (75)	18
$80 < x \leq 90$(85)	13
$90 < x \leq 100$(95)	8

$$Mean = \frac{\Sigma f(x)}{\Sigma f} = \frac{15(55) + 28(65) + 18(75) + 13(85) + 8(95)}{15 + 28 + 18 + 13 + 8} = 71\frac{19}{41}$$

$$Square\ of\ Mean = \left(\frac{\Sigma fx}{\Sigma f}\right)^2 = \left(71\frac{19}{41}\right)^2 = 5107\frac{33}{1681}$$

$$Mean\ of\ Square = \frac{\Sigma f(x^2)}{\Sigma f} = \frac{15(55)^2 + 28(65)^2 + 18(75)^2 + 13(85)^2 + 8(95)^2}{15 + 28 + 18 + 13 + 8}$$
$$= 5256\frac{29}{41}$$

Standard Deviation $= \sqrt{\frac{\Sigma f(x^2)}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{5256\frac{29}{41} - 5107\frac{33}{1681}} = 12.2\ (3sf)$

Interpretation of Standard Deviation Values

If you are being asked to compare two sets of data, for which both data have a different mean or standard deviation, here is a list of helpful words for students to use. However, the table below is non-exhaustive, and you should always consider the context of the question and make use of common sense, before trying to answer the question.

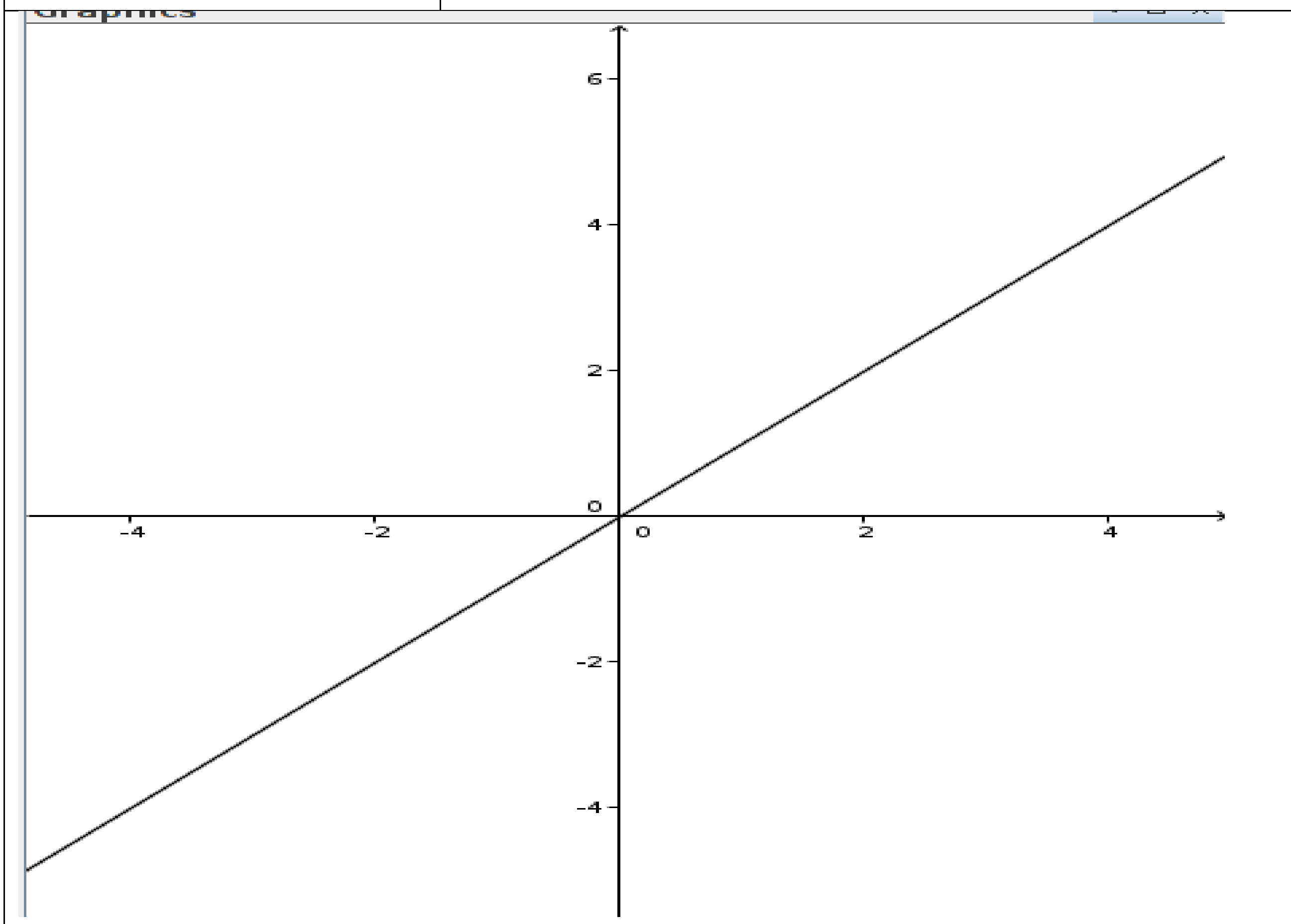
	Higher Mean	**Lower Mean**	**Higher Standard Deviation**	**Lower Standard Deviation**	
Generic Situation	The dataset generally contains higher value.	The dataset generally contains lower value.	The dataset values are generally more spread out.	The dataset values are generally less spread out.	
Pricing	More Expensive in General	Less Expensive in General	Value of Prices more spread out	Value of Prices less spread out	
Performance/Scores	Perform Better in General	Perform Worse in General	Scores are less Consistent	Scores are more Consistent	
Timing (To Complete a Task)	Slower	Faster	Less Consistent	More Consistent	
Resource Usage (To Complete a Task)	Less Efficient	More Efficient	Less Consistent	More Consistent	
Accuracy Matters	More Accurate	Less Accurate	Less Consistent	More Consistent	

Title	Shapes of Graphs of Commonly Seen Power Function
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	4/10/2018

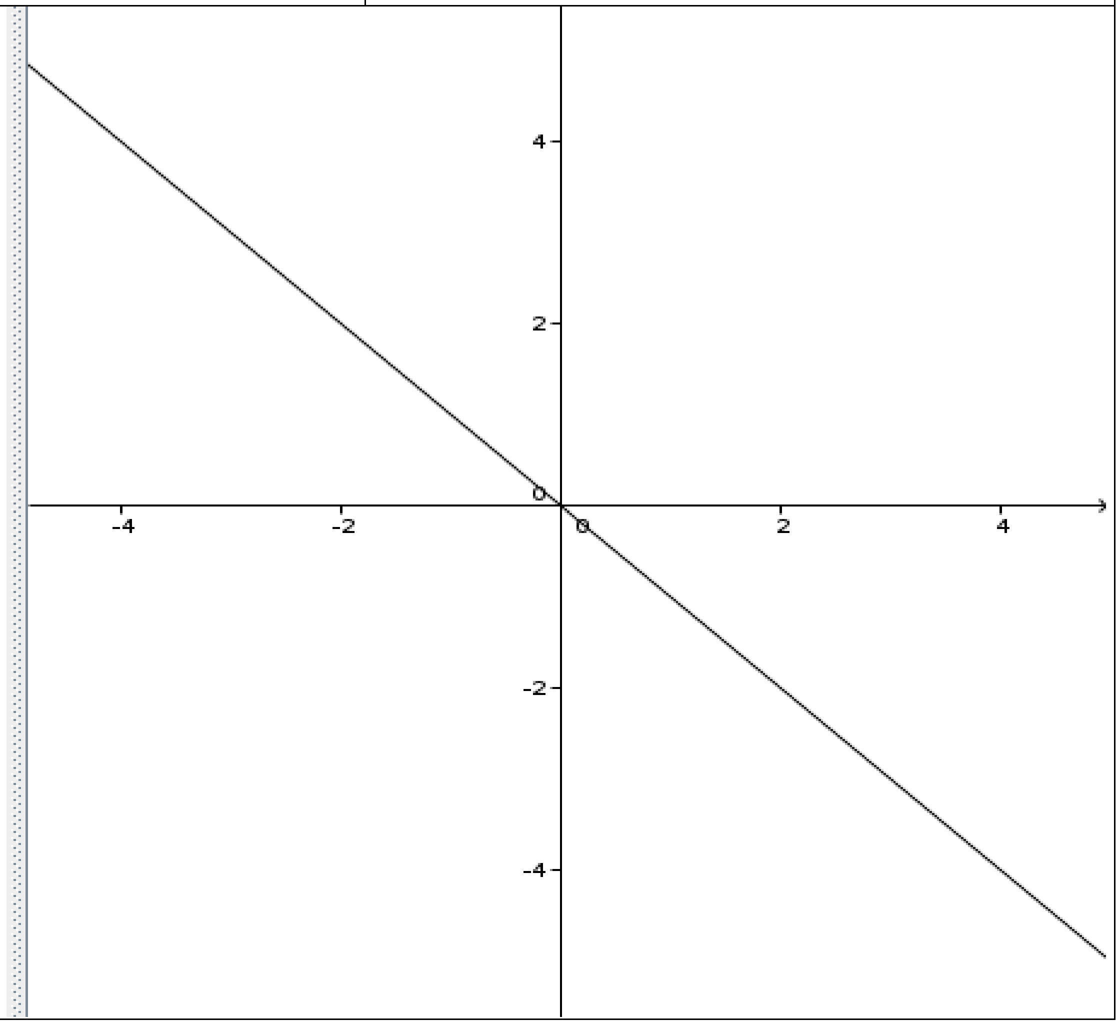
This topic requires a bit of memory work. I am showing you all the **generic** shapes of graphs of power function.

(Note: **In exam, please specify your X-axis and Y-axis and write down the equation of graph beside the line you drew, or marks will be deducted**. The computer software I used to generate these graphs doesn't do that for me, as it is widely understood in the computer software manuals that horizontal-axis are called X-axis while vertical-axis are as defined Y-axis.)

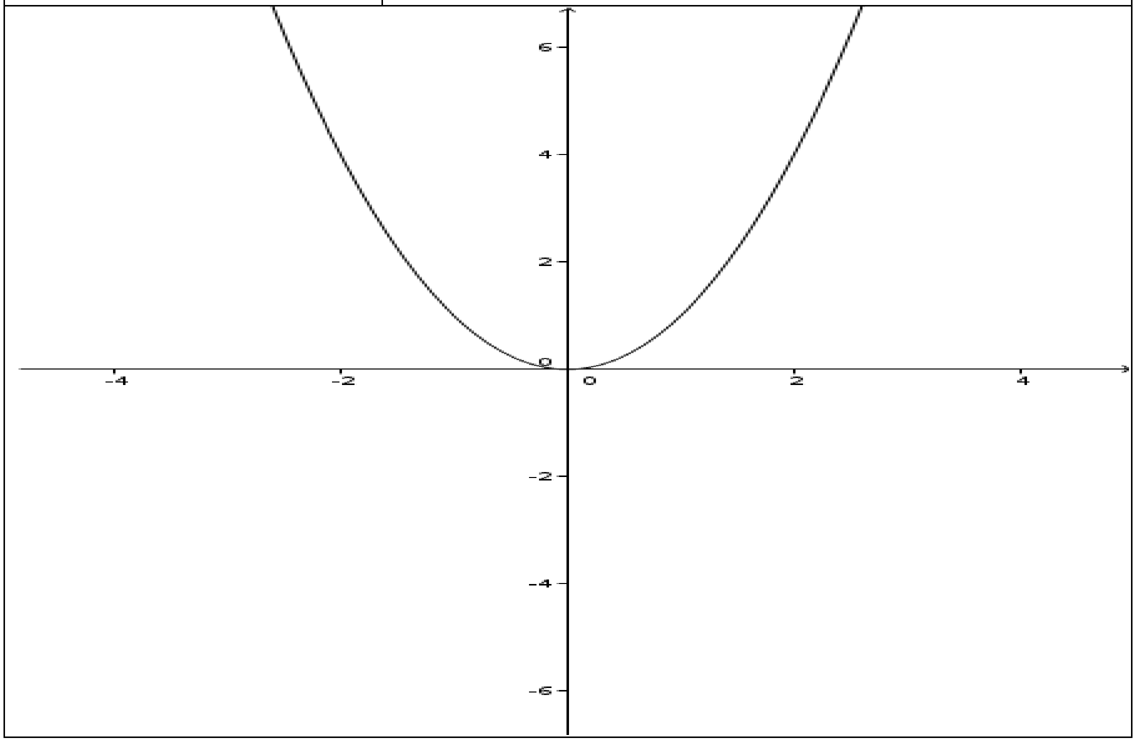
Equation	$y = mx + c$ (for $m > 0$)
Name	Linear Function



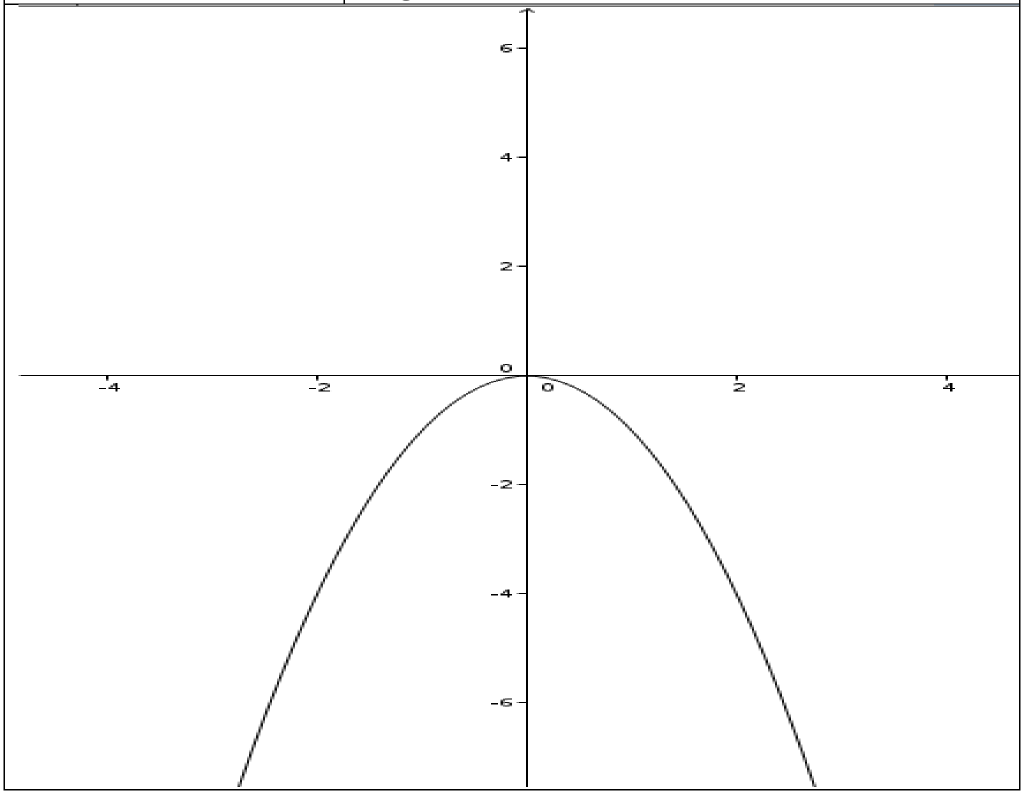
Equation	$y = mx + c$ (for $m < 0$)
Name	Negative Linear Function



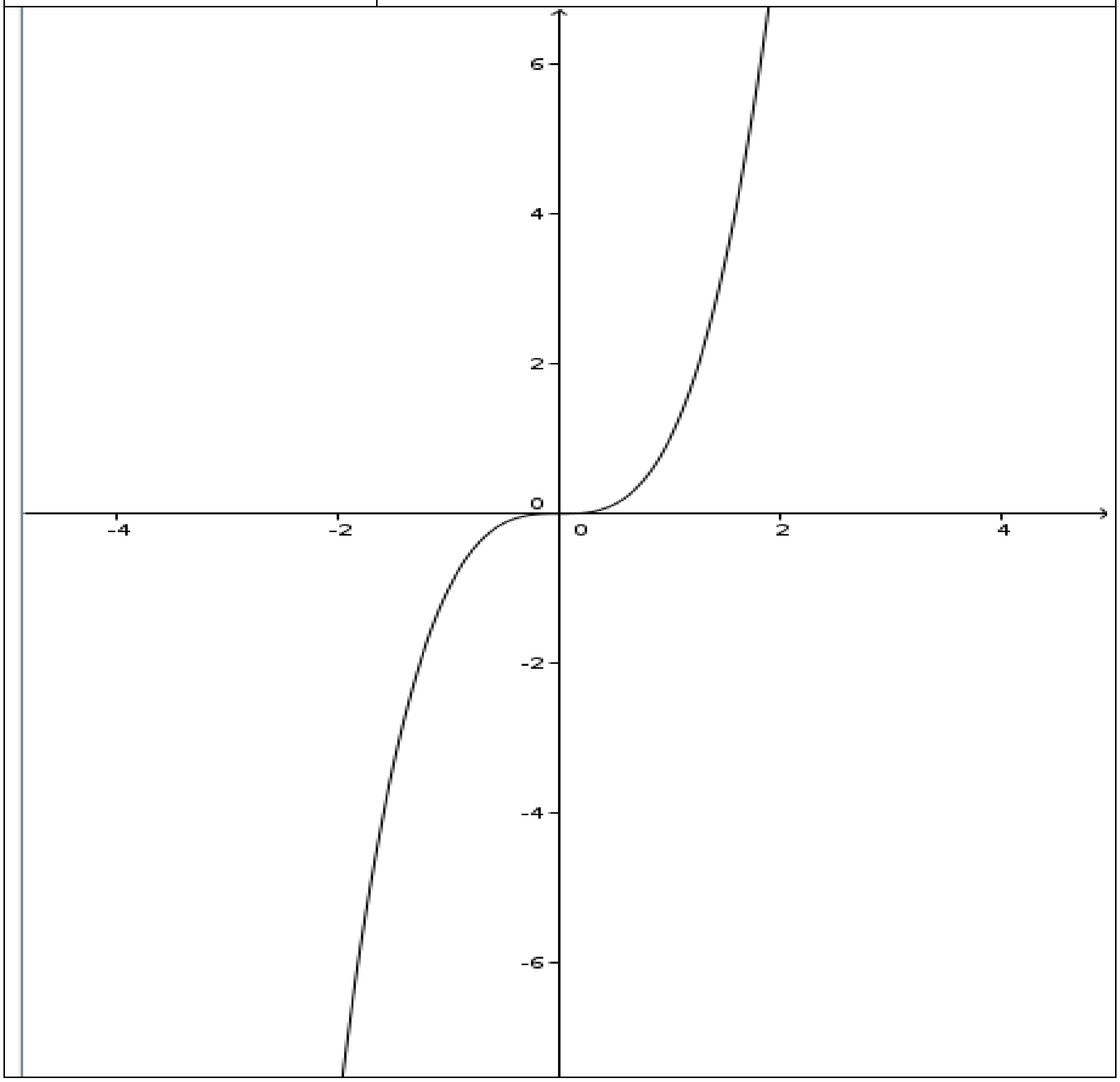
Equation	$y = ax^2 + bx + c$ (for $a > 0$)
Name	Quadratic Function



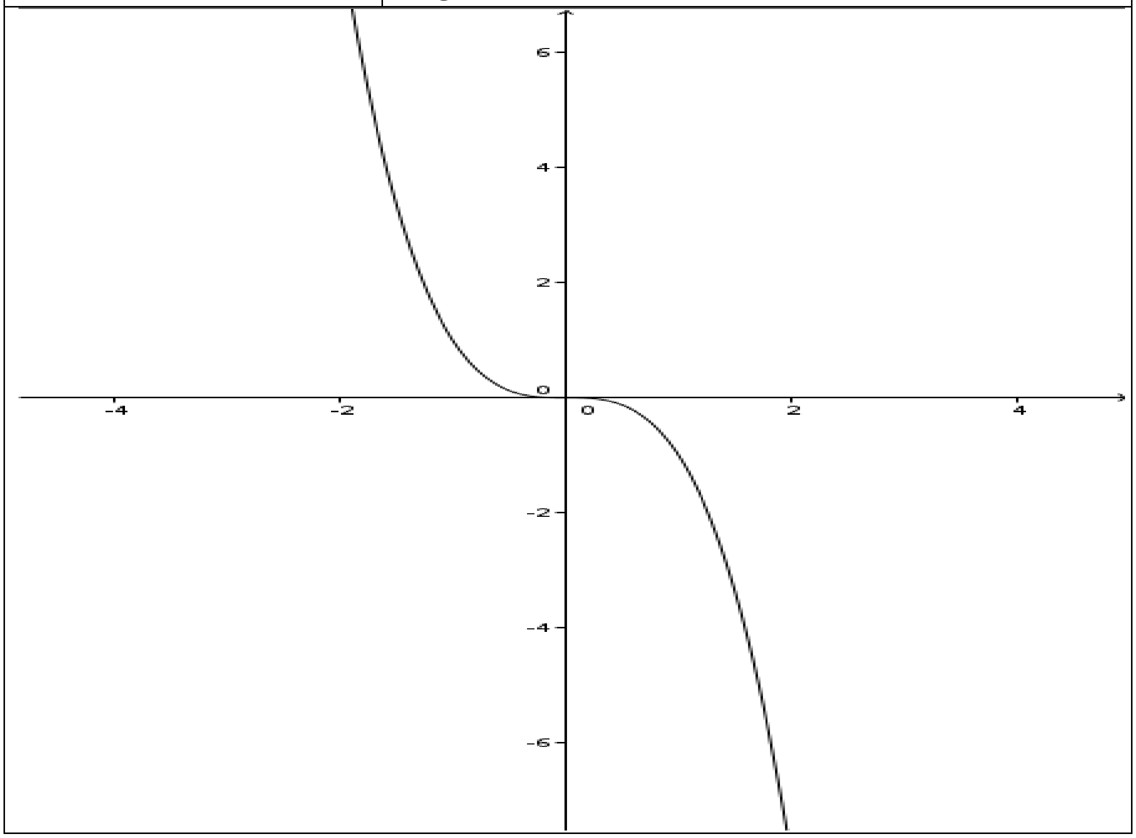
Equation	$y = ax^2 + bx + c$ (for $a < 0$)
Name	Negative Quadratic Function



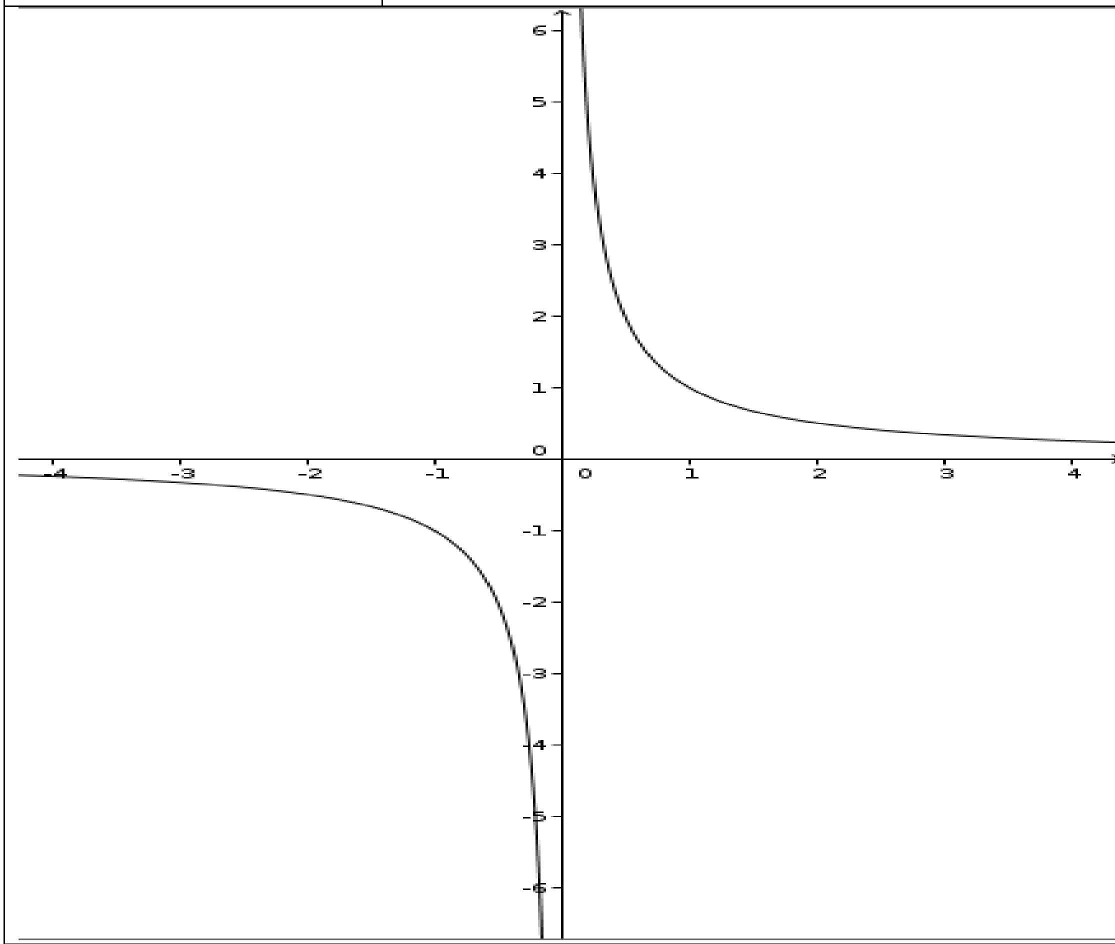
Equation	$y = ax^3 + bx^2 + cx + d$ (for $a > 0$)
Name	Cubic Function



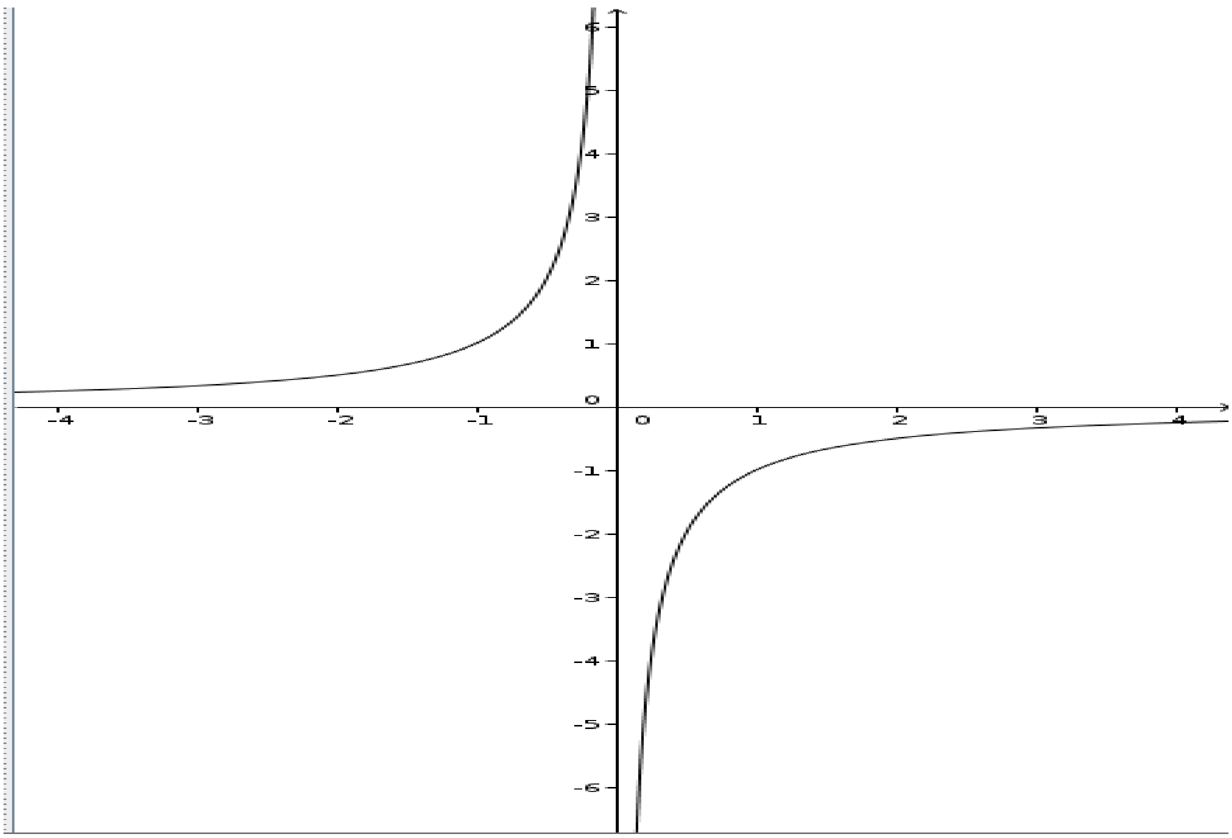
Equation	$y = ax^3 + bx^2 + cx + d$ (for $a < 0$)
Name	Negative Cubic Function



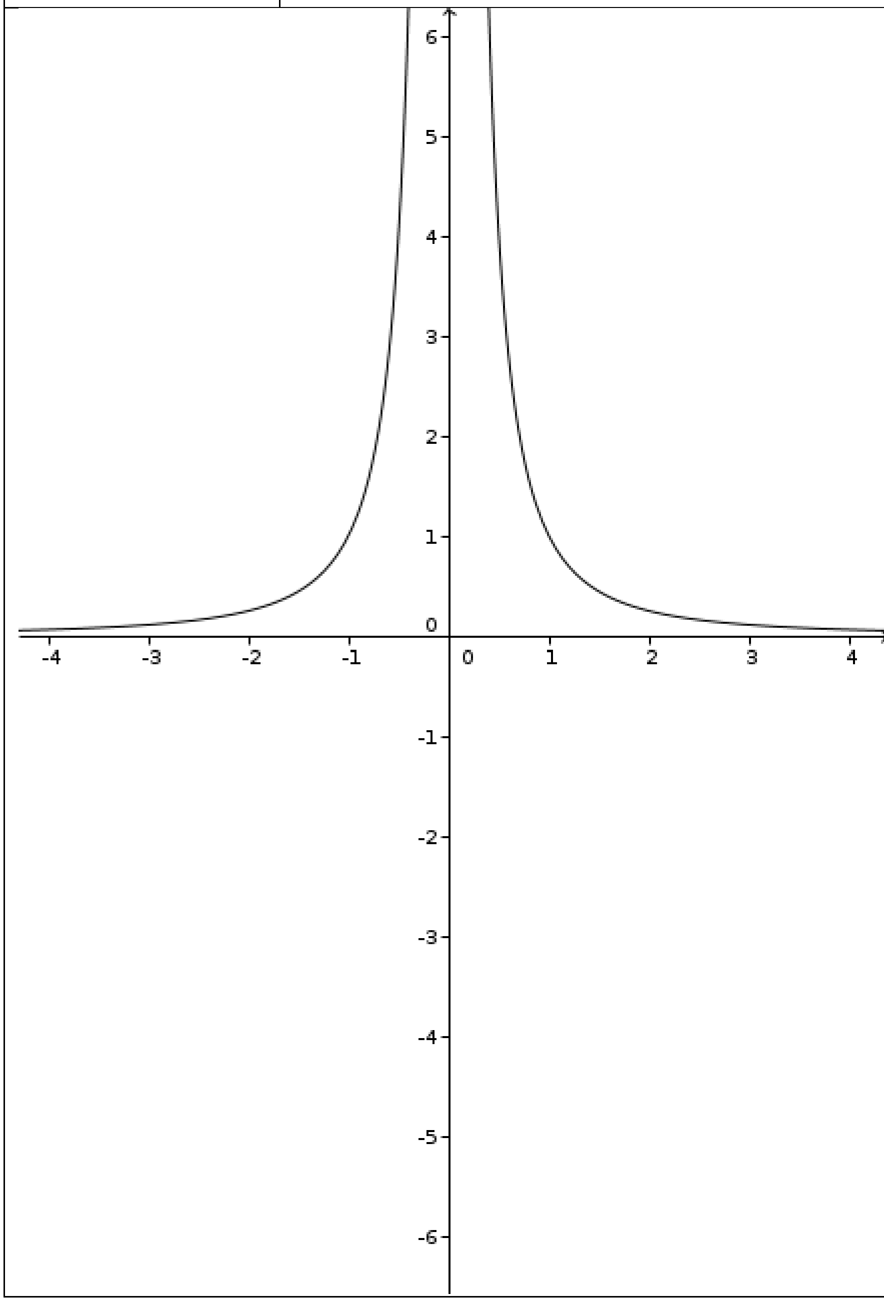
Equation	$y = \frac{a}{x}$ $Or\ y = ax^{-1}$ (For $a > 0$)
Name	Linear Reciprocal Function ***Take note that the graph will not have any contact with the Y-axis or X-axis.



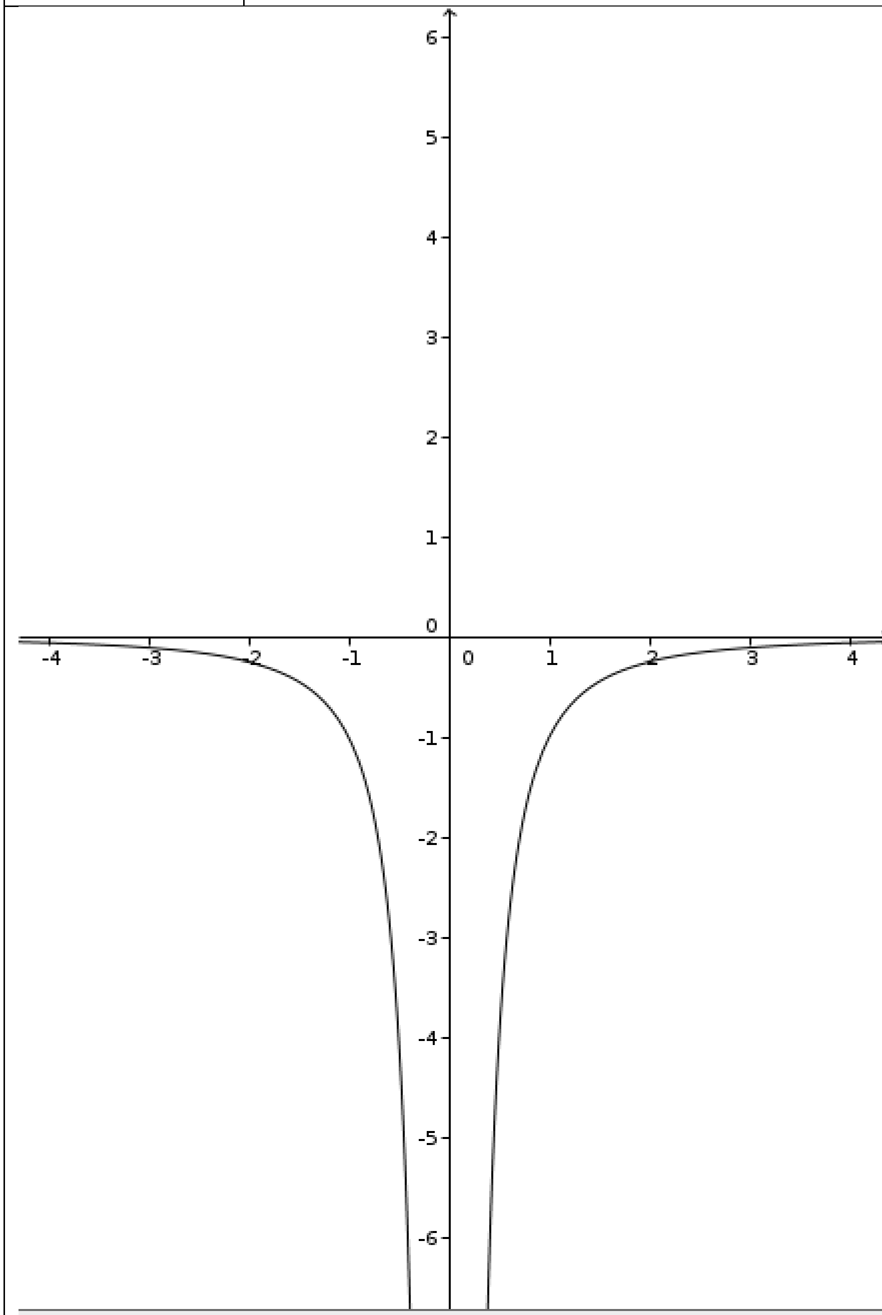
Equation	$y = \frac{a}{x}\ OR\ y = ax^{-1}$ (for $a < 0$)
Name	Negative Linear Reciprocal Function ***Take note that the graph will not have any contact with the Y-axis or X-axis.



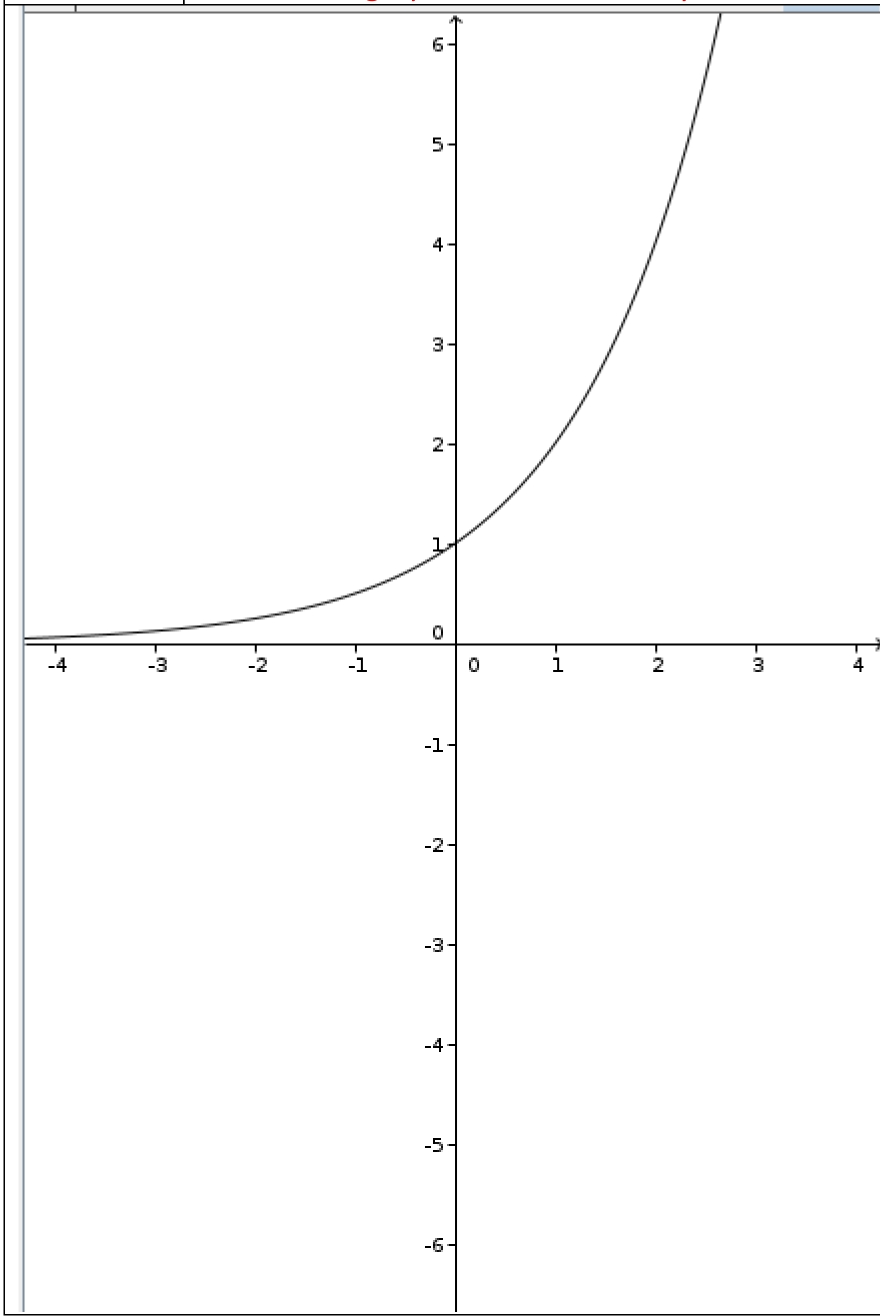
Equation	$y = \frac{a}{x^2}\ OR\ y = ax^{-2}$(for $a > 0$)
Name	Quadratic Reciprocal Function ***Take note that the graph will not have any contact with the Y-axis or X-axis.



Equation	$y = \frac{a}{x^2}\ OR\ y = ax^{-2}$(for $a < 0$)
Name	Negative Quadratic Reciprocal ***Take note that the graph will not have any contact with the Y-axis or X-axis.



Equation	$y = a^x$ $(\boldsymbol{a} > \mathbf{1})$
Name	Exponential Function **Note the graph doesn't have any contact with the X-axis



Title	**Solving Problem Sums with Quadratic Equations**
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [From Nanyang Polytechnic Mentoring Club]
Editor	Hui Ling, Ngee Ann Polytechnic
Inspector	Lee Jian Lian
Date	26/2/2018

I am Wang Sheng from Mentoring Club. I am here to demonstrate how do you solve a problem sum by using quadratic equation. The procedure if you notice, it is standardized among all types of questions requiring you to form quadratic equation and solve them to get your answer.

[DO NOT READ THIS DOCUMENT IF YOU DON'T UNDERTAND HOW TO FIND SOLUTIONS TO A QUADRATIC EQUATION. THIS DOCUMENT ASSUMES YOU UNDERSTAND COMPLETING THE SQUARES AND QUADRATIC FORMULA.]

Step	Description
1	Form Expressions
2	Form Equation
3	Reduce the Equation to Quadratic Form
4	Find the Solutions to the Quadratic Equation(s) derived
5	Reject Values that doesn't make sense. (Examples: Dividing by Zero, Negative Distance.) [If Applicable]
6	Answer Remaining Questions

The six steps will be demonstrated in the next page and these are based on my teaching experience in my CCA. I will try to make the whole document as simple to understand as possible for beginners.

[Note: Questions Taken from CASCO Mathematics Assessment Book 4B]

Example Question 1:

A motorist travelled 60km from P to Q at an average speed of x km/h.

(a) Write down an expression, in terms of x, for the time taken, in hours, for the journey.

On his return journey from Q to P, his average speed was reduced by 5 km/h due to heavy traffic on the way.

(b) Write down an expression, in terms of x, for the time taken, in hours, for the return journey.

(c) If the return journey took 10 minutes longer, form an equation in x and show that it reduces to

$$x^2 - 5x - 1800 = 0$$

(d) Solve the equation $x^2 - 5x - 1800 = 0$

(e) Find the time taken for the return journey.

Step 1. Form Expression

Q1(a)	Distance = 60km Average Speed = x km/h Since $\frac{distance}{speed} = time$ Time taken from P to Q must be = $\frac{60}{x}$ $hours$.
Q1(b)	Distance = 60 km (Reverse Direction Thus Distance Unchanged) Speed = $x - 5$ km/h. Since $\frac{distance}{speed} = time$ Time taken from Q to P must be = $\frac{60}{x-5}$ hours.

Step 2. Form Equation

Q1(c)	Since the return journey is 10 minutes longer, and the unit specified in the question is in hours, we must convert "10 minutes" into $\frac{1}{6}$ hours. Therefore, the following equation is formed. (Take note, the return journey is longer, so the value must be, return journey – the initial trip = $\frac{1}{6}$ hours. $$\frac{60}{x-5} - \frac{60}{x} = \frac{1}{6}$$

Step 3. Show that the equation can be reduced into quadratic form as shown in the question.

<table>
<tr><td>Q1(c)</td><td>

$$\frac{60}{x-5} - \frac{60}{x} = \frac{1}{6}$$

$$\frac{60(x)}{x(x-5)} - \frac{60(x-5)}{x(x-5)} = \frac{1}{6}$$

$$\frac{60x - 60x + 300}{x^2 - 5x} = \frac{1}{6}$$

$$\frac{(60x - 60x + 300)}{x^2 - 5x} = \frac{1}{6}$$

$$\frac{300}{x^2 - 5x} = \frac{1}{6}$$

$$300 = \frac{1}{6}(x^2 - 5x)$$

$$6(300) = x^2 - 5x$$

$$0 = x^2 - 5x - 1800$$

$x^2 - 5x - 1800 = 0$ [Shown]

</td></tr>
<tr><td>Notes</td><td>TAKE NOTE OF NEGATIVE SIGNS</td></tr>
</table>

Step 4. Find Solution to the Equation (Also called solve the equation)

<table>
<tr><td>Q1(d)</td><td>

(In this case, equation will be solved using quadratic formula method, if the question doesn’t specify any methods, you can use any method you like.)
However, if the question does require specific methods to be used to solve the equation, use the stated method in the question.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$a = 1, b = -5, c = -1800$$

</td></tr>
</table>

	$x = \frac{-(-5) + \sqrt{(-5)^2 - 4(1)(-1800)}}{2(1)}$ OR $x = \frac{-(-5) - \sqrt{(-5)^2 - 4(1)(-1800)}}{2(1)}$ $x = 45\ OR\ x = -40$

Q1(e)	Since you can't have negative speed value, $x = -40$ must be rejected. Since question asks for time required for return journey. They are asking for the value of $\frac{60}{x-5}$ Substitute $x = 45$ into the above expression to get $\frac{60}{(45-5)} = 1.5$ Time taken for return journey = $1\ hour\ and\ 30\ minutes$.

Title	Laws of Indices – Evaluation of Algebraic Expressions Containing Powers and Simplification of Expressions)
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	25/12/2018

Objective of document

- Explanation of Various Laws of Indices (That some school teachers miss out)
- Demonstration on questions which involves complicated expression involving powers, fraction inside fraction, negative powers and multiple brackets in a single expression.

Basic Laws of Indices	Demonstration of Concept Using Numbers
$a^m(a^n) = a^{m+m}$	$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2) = 2^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^1} = \frac{2 \times 2 \times 2}{2} = 2^{3-1} = 2^2$
$(a^m)^n = (a)^{mn}$	$(2^2)^3 = (2 \times 2)^3$ $= (2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^{2(3)} = 2^6$
$a^{-n} = \frac{1}{a^n}$ (Note: denominator must not equal zero as division by zero is undefined)	Using the previously mentioned law of indices, we can write an expression: $\frac{2^2}{2^4}$ $\frac{2^2}{2^4} = \frac{2\times2}{2\times2\times2\times2} = 2^{2-4} = 2^{-2} = \frac{1}{2^2}$
$a^0 = 1$ (Note $a \neq 0$, as 0^0 is undefined)	Using the previously mentioned law of indices, we can write an expression $\frac{2^3}{2^3} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2} = 2^{3-3} = 2^0 = 1$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (Note $b \neq 0$, as division by zero is undefined)	$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2^3}{3^3}$
$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	Using the previously mentioned law of indices, we can write the following: $2^4 = 2 \times 2 \times 2 \times 2$ $2^{\frac{4}{2}} = \sqrt{2 \times 2 \times 2 \times 2} = \sqrt{2^4} = 2^{\frac{4}{2}} = 2^2$
$(ab)^n = a^n(b^n)$	

Derived Law of Indices	Simplest Proof of Concept
$\left(\frac{a^m}{a^n}\right)^{-1} = \frac{a^n}{a^m}$	$\left(\frac{1}{5}\right)^{-1} = \frac{1}{\left(\frac{1}{5}\right)} = \frac{5}{1} = 5$
$\left(\frac{a^m}{a^n}\right)^{-x} = \left(\frac{a^n}{a^m}\right)^{x}$	

Tips to do complicated indices question (with nested fractions, powers and negative powers all in the same question.)

Read question (Examine the details closely)
Take note of any "out of the ordinary" situation (Example can include zero powers, that makes solving a seamlessly complicated question easier)
Take note of any negative powers.
Take note of any roots.
Write the expression in fully index notation
Solve the question from inner brackets to outer bracket
Check your answers

I will demonstrate some questions and explain how the tips can be applied.

(Questions all taken from CASCO Mathematics Assessment Book 4B)

Example 1:

Simplify $(4x^2)^{\frac{3}{2}}$

$(4x^2)^{\frac{3}{2}}$	(Things to take note highlighted)
$= 4^{\frac{3}{2}}x^{2\left(\frac{3}{2}\right)}$	Apply the following laws of indices $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $(ab)^n = a^n b^n$
$= 8x^3$	(Final Answers)

Example 2:

Evaluate $\left(\frac{1}{2}\right)^{-4} \times 8^0$

$\left(\frac{1}{2}\right)^{-4} \times 8^0$	Things to take note (Zero Power and Negative Powers)
$= \left(\frac{2}{1}\right)^{4} \times 1$	Apply the following law of indices $a^0 = 1$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
$= 16$	

Example 3:

Given that $\frac{(x^2)^3 \times \sqrt{x}}{\sqrt[3]{x}} = x^n$. Calculate the value of n.

$\frac{(x^2)^3 \times x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^n$	Convert values all to index notation
$\frac{(x^6 \times x^{\frac{1}{2}})}{x^{\frac{1}{3}}} = x^n$ $\frac{x^{6+\frac{1}{2}}}{x^{\frac{1}{3}}} = x^n$	Apply the following law of indices $(a^m)^n = a^{mn}$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
$x^{6\frac{1}{2}-\frac{1}{3}} = x^{6\frac{1}{6}} = x^n$	Apply the following law of indices $\frac{a^m}{a^n} = a^{m-n}$

$n = 6\frac{1}{6}$	

Title	Venn Diagrams and Set Language
Author	Lim Wang Sheng, School of Information Technology, Nanyang Polytechnic [CCA: NYP Mentoring Club]
Date	21/7/2018

Knowing a lot of students are having problems with this topic, I will also be explaining the tricks I've used (derived from my secondary school teacher's approach) to do questions pertaining to this topic. Before we began, let's talk about why this topic is important.

It serves as a basic concept to be applied to various other concepts in the following fields

- Electronics (Logic Gate and Computer System Programming)
- Software Development (Creating Computer Software and Understanding Software Logic)
- Mathematical Proofs and Logic Demonstration

Notation	Alternative Notation	Meaning of the Notation
ε	$\mathbb{U}$ (Not used in 'O' Levels, appears in other textbooks)	Universal Set: A set containing all the possible element, in which all are subsets.
$\cap$ (E.g. $A \cap B$)	AND (Not used in 'O' Levels, appears in programmer guide and electrical theory books)	Intersection: To take the intersection between multiple sets. Further explanation: When you say to take the intersection between multiple sets, you are taking any elements common between the sets. Example Below Meant: To take the intersection or common elements between A and B.
$\cup$ (E.g. $A \cup B$)	OR (Not used in 'O' Levels, appears in programmer guide and electrical theory books)	Union: To take element that appear in either set, along with the intersection between the sets. Further explanation: When you say to take the union between multiple sets, you are taking every possible element in both sets, along with the intersection. Example Below Meant: To take the elements that appear in either Set A or Set B, along with the intersection between A and B.
$\in$		Element of: To explain that something is an element of a set.
$n(A)$		Number of Element in a specific set: In this case, we are talking number of elements in set A.
A'	A^c (Not used in 'O' Levels, appears in other textbooks)	Complement of a specific set: In this case, we are talking about the complement of set A, or simply, whatever element not found in set A.

$\subseteq$ E.g. $(A \subseteq B)$		To be a subset of: In the example below the notation, we are saying that whatever element found in set A is found in set B as well.
$\subset$ E.g. $(A \subset B)$		To be a proper subset of: In the example below the notation, we are implying that whatever element found in set A is found in set B. However, <u>set B must contain at least one element not found in set A.</u>
$\{\}$ E.g. $A = \{1,2,3\}$		To contain the following elements as mentioned. In the example, we meant set A is to contain elements 1,2,3.
$\emptyset$ E.g. $A \cap B = \emptyset$		To describe a null set, or a set that doesn't exist. In the example, it implies the intersection between set A and set B does not exist.

Example 1. Drawing Venn Diagram for a Given Question. (Elements of sets not given, just ask you to draw Venn Diagram)

A universal set is given by ε and its subsets are given by A and B.
Draw separate Venn Diagrams, for the following situations.

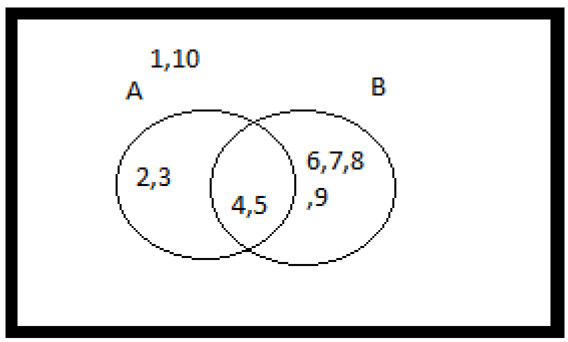
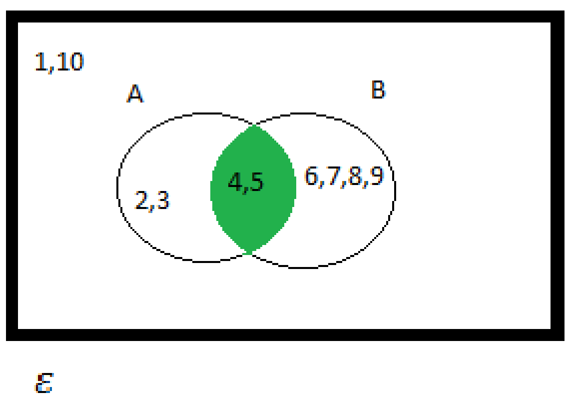
(a) $A \cap B$
(b) $A \cup B$
(c) $A' \cup B$
(d)$(A \cap B')'$
(e) $A \cap B = \emptyset$

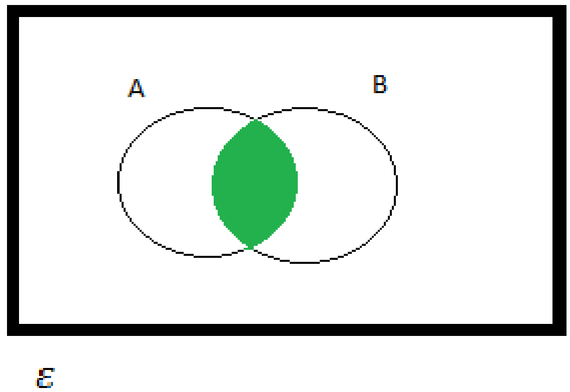
To facilitate understanding of the question, I would assign "dummy elements" to set A, set B and ε. The dummy element will not be used for answering of question. In an exam situation, I will either ask for extra paper and write the dummy elements down and do workings with the help of the dummy element or, I will do workings with a pencil with the dummy elements and erase them later.

Dummy Elements
$\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$
$A = \{2,3,4,5\}$
$B = \{4,5,6,7,8,9\}$

Example 1(a)

<table>
<tr><td>Step 1: Understand the question using the dummy elements you produce.</td><td>$A = \{2,3,4,5\}$
$B = \{4,5,6,7,8,9\}$
$\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$

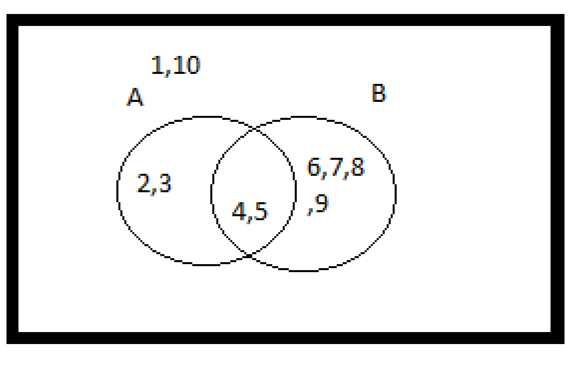
In this case, they are asking for $A \cap B$. We are going to look for dummy elements common to both A and B.</td></tr>
<tr><td>Step 2: Draw a Venn Diagram, in pencil, along with the dummy elements into the diagram.</td><td>
</td></tr>
<tr><td>Step 3: Shade the area enclosed within the dummy elements you are finding, in this case, we shade the area that encloses 4,5 as follows.</td><td>
</td></tr>
</table>

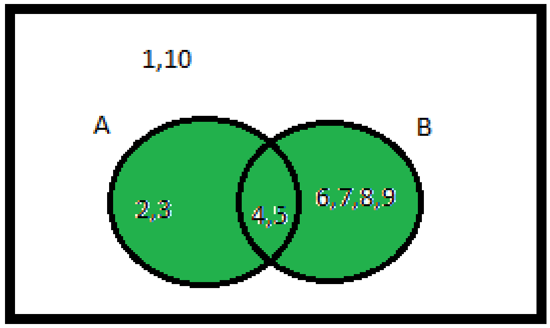
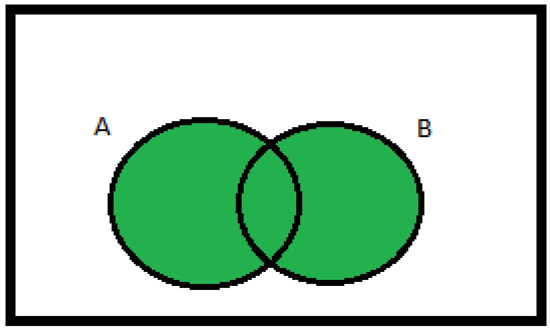
<table>
<tr><td>Step 4: Carefully Erase the Dummy Elements while retaining the shaded area as follows.</td><td>
</td></tr>
</table>

Example 1(b)
Same procedure will apply as well, except you are shading the union, meaning element A, element B, along with the intersection between A and B. since the question asked for $A \cup B$.

Example 1(b)

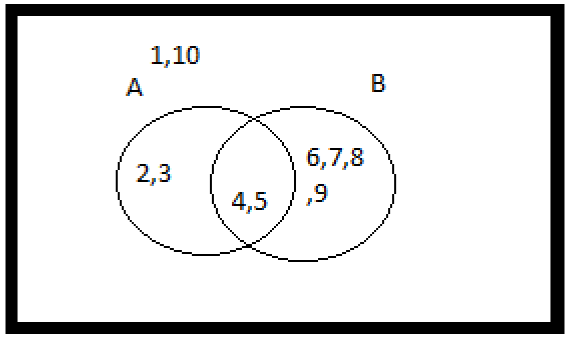
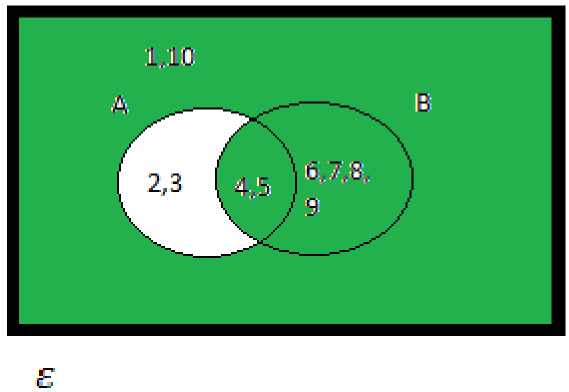
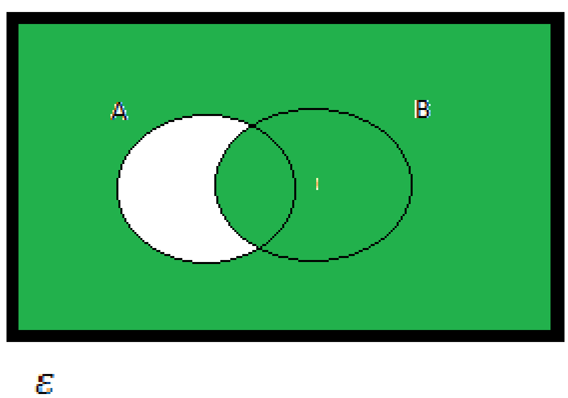
<table>
<tr><td>Step 1
Understand the question using dummy elements</td><td>$A = \{2,3,4,5\}$
$B = \{4,5,6,7,8,9\}$
$\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$

In this case, they are asking for $A \cup B$, the union between both sets, which means, elements in both A and B, along with elements they share should be highlighted as above.</td></tr>
<tr><td>Step 2: Draw a Venn Diagram, in pencil, along with the dummy elements into the diagram.</td><td>
</td></tr>
</table>

Step 3: Shade area which encloses the dummy elements highlighted previously, in this case, we shade the area occupying 2,3,4,5,6,7,8,9	
Step 4: Erase all dummy elements you written previously carefully.	

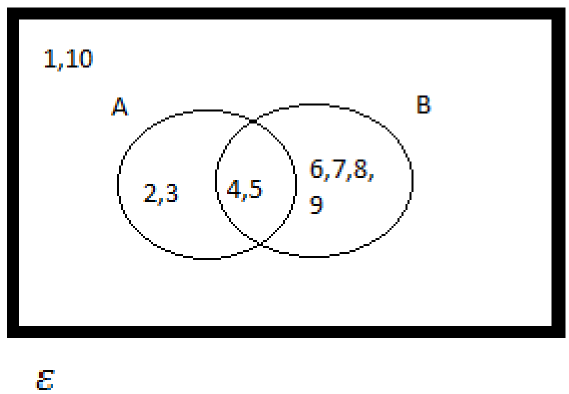
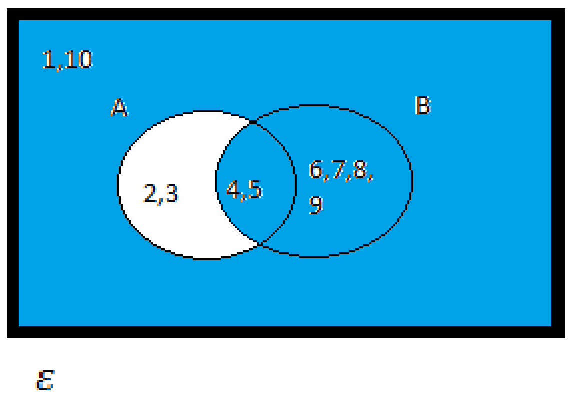
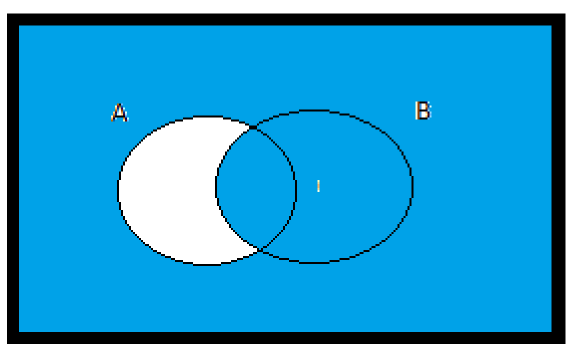
Example 1(c)

Step 1: Understand the question using dummy elements	$\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$ $A = \{2,3,4,5\}$ $B = \{4,5,6,7,8,9\}$ Since the question asked for $A' \cup B$, we highlighted elements that satisfy the following conditions. - Found in universal set - Not found in A - Found in B

Step 2: Draw a Venn Diagram, in pencil, along with the dummy elements into the diagram.	
Step 3: Shade areas which encloses the elements highlighted previously.	
Step 4: Erase Dummy Elements carefully.	

Example 1(d)

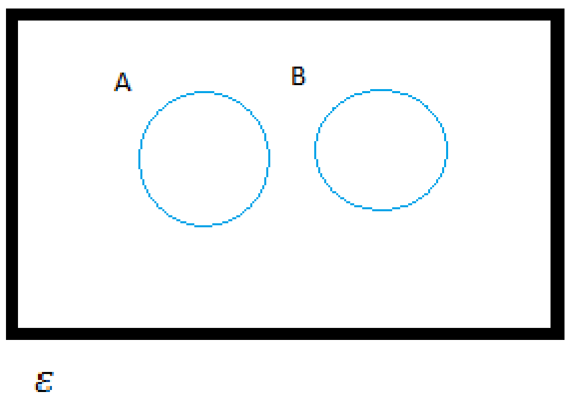
Step 1: Understand the question using the dummy elements you produce.	$\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$ $A = \{2,3,4,5\}$ $B = \{4,5,6,7,8,9\}$ Question Asked for $(A \cap B')'$. We have analyses the question in following detail. $A \cap B'$ is to be highlighted as follows: $\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$ $A = \{2,3,4,5\}$ $B = \{4,5,6,7,8,9\}$ $B' = \{1,2,3,10)$ Question asked for $(A \cap B')'$. Values highlighted in the previous part will not be highlighted in the next part of the working. Values not highlighted will be highlighted in the next part of the working like this. $\varepsilon = \{1,2,3,4,5,6,7,8,9,10\}$ $A = \{2,3,4,5\}$ $B = \{4,5,6,7,8,9\}$

Step 2: Draw a Venn Diagram, in pencil, along with the dummy elements into the diagram.	
Step 3: Shade areas which encloses the elements highlighted previously.	Areas enclosing the following elements will be shaded. $\{1,4,5,6,7,8,9,10\}$ 
Step 4 Erase Dummy Elements carefully to get the following	

Example 1(e) [Special Case]
*** Please commit this to memory.

Firstly, we need to understand what set notation $A \cap B = \emptyset$ mean.

We break this down into several parts

$A \cap B$	Intersect Between A and B
$\emptyset$	Null Set (In English Terms, doesn't exist.)
$A \cap B = \emptyset$	The intersection between A and B is non-existent, implying set A and set B doesn't overlap in a Venn Diagram. This is also known as "disjointed sets".
	Drawn out in Venn Diagram Form. 
	***Unless question specify additional instructions, you most likely don't have to shade the Venn Diagram, ask your teachers if in doubt as my school doesn't require us to shade diagram if sets are supposed to be disjointed.

Title	Basic Trigonometry and Geometry of Congruent & Similar Triangles
Notes	This article assumes that you have already studied and understand the basic rules of geometry as it will include basic level guides to proving congruence and similarity of triangles.

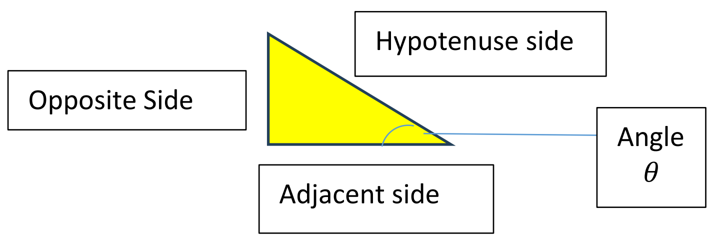
Pythagoras' Theorem
Given that c refers to the hypotenuse (the longest side) of a right-angled triangle, a and b represent the other 2 sides of a right-angled triangle, the Pythagoras' theorem implies the following
$a^2 + b^2 = c^2$
Which can also be rewritten as $\sqrt{a^2 + b^2} = c$

Basic Trigonometric Ratios of Right-angled Triangle

$$\sin\theta = \frac{Opposite\ Side}{Hypotenuse\ Side}$$

$$\cos\theta = \frac{Adjacent\ Side}{Hypotenuse\ Side}$$

$$\tan\theta = \frac{Opposite\ Side}{Adjacent\ Side}$$



In the above right-angled triangle, we can see that the hypotenuse side is the longest in any right-angled triangle. The adjacent side refers to the side of the triangle that is closest to the angle in question. Finally, the opposite side refers to the side of the triangle that is opposite of the given angle θ in question.

Uses of Basic Inverse Trigonometric Functions on Calculator

$$\theta = \sin^{-1}(\sin\theta)$$

$$\theta = \cos^{-1}(\cos\theta)$$

$$\theta = \tan^{-1}(\tan\theta)$$

Given any triangle, such as the triangle on the right of this row as an example.

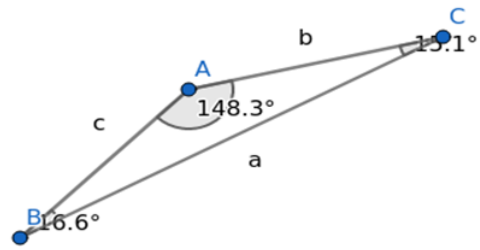
Sine Rule states the following:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Where $\angle A$ is opposite of side a
$\angle B$ is opposite of side b
$\angle C$ is opposite of side c

Or in the case of the triangle we have on the right side of the row

$$\frac{\sin \angle BAC}{a} = \frac{sin\angle ABC}{b} = \frac{\sin \angle BCA}{c}$$



Cosine Rule states the following:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

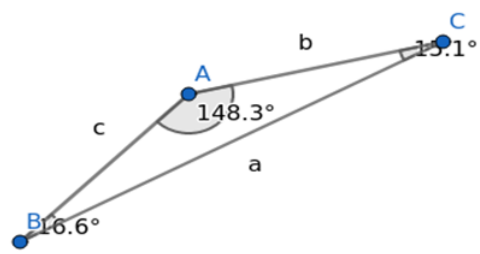
$$b^2 = a^2 + c^2 - 2ac\cos B$$

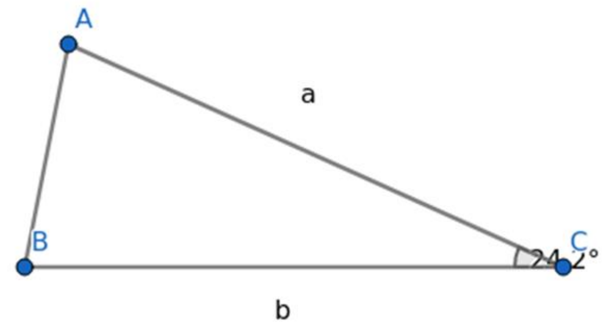
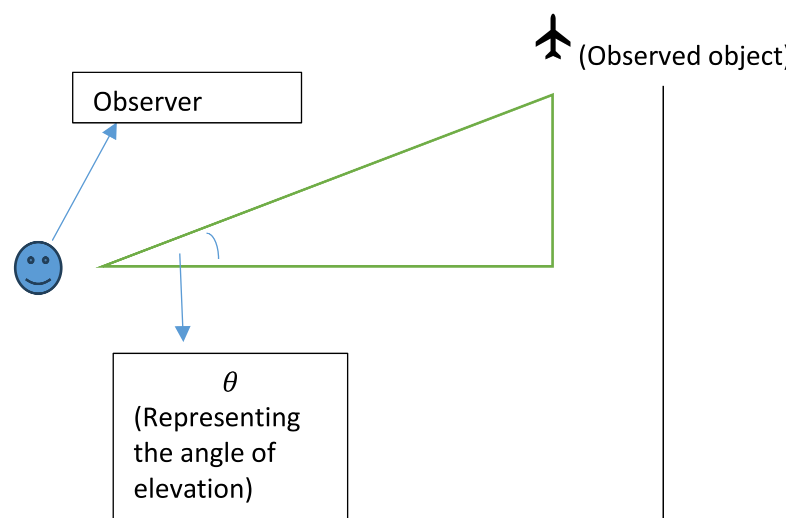
$$c^2 = a^2 + b^2 - 2ab\cos C$$

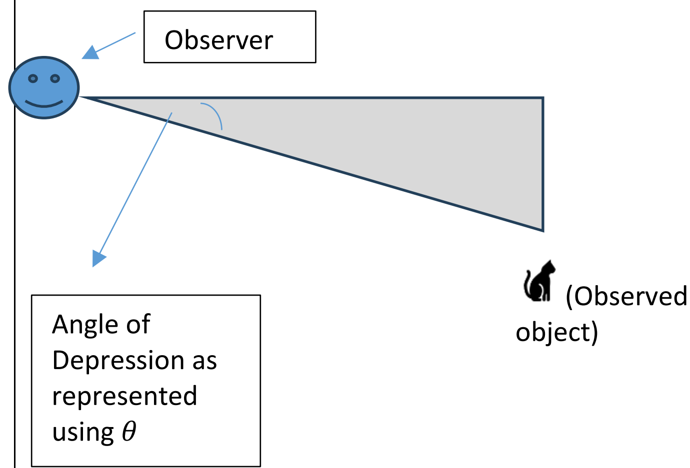
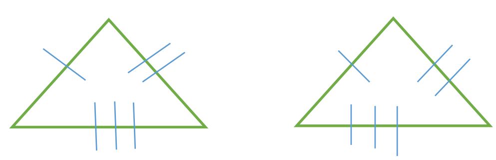
Where $\angle A$ is opposite of side a
$\angle B$ is opposite of side b
$\angle C$ is opposite of side c

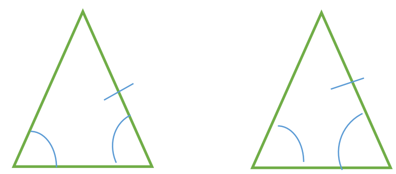
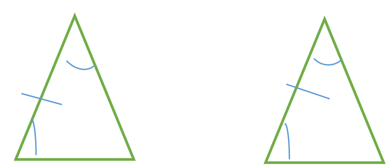
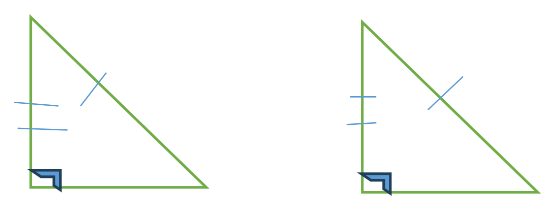
Because of the formula, it is also possible to find $\angle A, \angle B$ and $\angle C$ by working with the formula of cosine rule via the following as derived from the above formulae.

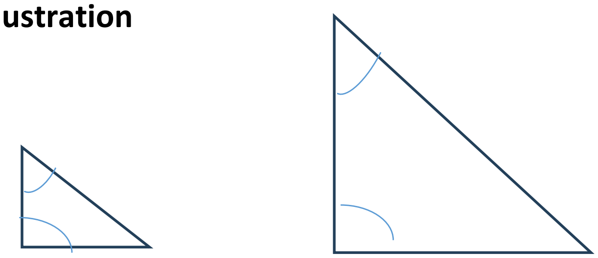
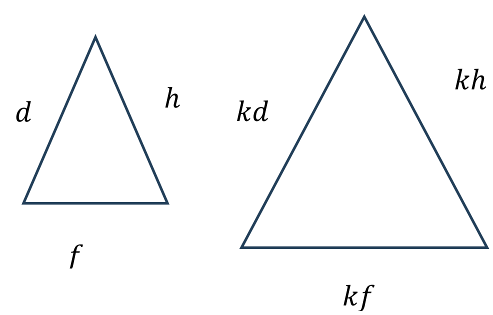
$$\cos A = \frac{a^2-b^2-c^2}{-2bc}$$

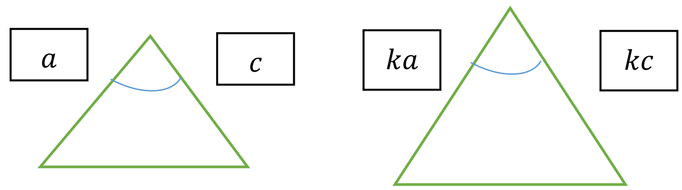


$\cos B = \frac{b^2-a^2-c^2}{-2ac}$ $\cos C = \frac{c^2-a^2-b^2}{-2ab}$ $\angle A = \cos^{-1}(\cos A)$ $\angle B = \cos^{-1}(\cos B)$ $\angle C = \cos^{-1}(\cos C)$	
Given a triangle, with 2 known sides length and 1 known angle that lies between the 2 known sides, the formula for area of such triangle is given as below: $\frac{1}{2}ab\sin C$	
Concepts involving Angle of Elavation and Angle of Depression Angle of Elavation	

Angle of Depression	
Concepts of Congruence of Triangles: Triangles are said to be congruent to each other when they are exactly the same shape, size and dimensions. **Notation:** We use the symbol "≡" to mean "is congruent to" and use the following rules to justify why two triangles are congruent.	**Example** **$\triangle ABC \equiv \triangle XYZ$ (By SAS Triangle Congruence Rule)** **Implies triangle ABC is congruent to triangle XYZ as both triangle share similar length on 2 sides and share the same included angle.**
SSS (Side-Side-Side) Congruence Rule Explanation – Two triangles are said to be congruent if all three corresponding sides are equal.	**Illustration of Concepts** 

SAS (Side-Angle-Side) Congruence Rule Explanation – Two triangle are said to be congruent if two corresponding side are equal and they have an equal including angle	
AAS (Angle Angle Side) Congruence Rule Explanation: Two triangles are said to be congruent when two angles and a corresponding side are equal	
ASA (Angle-Side-Angle) Congruence Rule Explanation: Two triangles are said to be congruent when two angle and an included side are equal	
RHS (Right-angle Hypothenus Side) Congruence Rule Explanation: Two triangles are said to be congruent when both triangle have a right angle, equal hypohtenus length and another equal side)	

Concept of Similarity in Triangles Both triangles are said to be similar to each other if and only both triangle share the same exact shape, but not necessarily be of the same size. When we try to state that two triangles are similar, we write the similar triangles in question, followed by the rule that justifies the statement.	**Example as follows** **$\triangle BCD$ is similar to $\triangle FGH$ (By AAA Similarity Rule)** **Implies triangle BCD is similar to triangle FGH because both triangles share at least two equal angles.**
AAA (Angle-Angle-Angle) Similarity Two triangles are said to be similar if all of three corresponding angle or just two corresponding angles are equal	**Illustration** 
SSS (Side-Side-Side Similarity) Two triangles are said to be similar if all of three corresponding sides of one triangle is of direct proprtion to the other	

SAS (Side-Angle-Side Similarity) Two triangles are said to be similar if two corresponding sides are of direct proportion to each other and an included angle is equal	

Title	Matrices – Rules, Operations and Applications
Notes and Credits	Some examples here are taken directly from CASCO Mathematics Tutor 4B for demonstration purposes
Date	

Before we proceed, I will be providing explanation on various vocabulary that surrounds how any matrices would be described, added, subtracted and multiplied, it is very important to understand these terms as any correct matrix operation relies heavily on correct understanding of core concepts.

Typical structure of a matrix as an example	
$\begin{pmatrix} 3 & 6 \\ 0 & 4 \end{pmatrix}$	As seen from the matrix on the left side of this table row, we see a typical matrix which consist of a large bracket with a set of numbers (also referred to as elements) in a rectangle array format.

The term rows and columns	
Rows	Refers to an arrangement of matrix elements made horizontally
Columns	Refers to an arrangement of matrix elements made vertically

Let's see a few real examples that would clear confusion instantly
For the following matrices, state the number of rows and column

$$\begin{pmatrix} 4 & 3 & 1 \\ 0 & 8 & 4 \end{pmatrix}$$

Referring to our definitions of rows and columns, we get the following idea on the above example

The matrix above has 2 elements each in horizontal format for every 3 elements arranged vertically, therefore the matrix above has 2 rows and 3 columns.

Example

$$\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}$$

Referring to our above example, we get the following idea

The matrix just right above has 3 elements in horizontal arrangement for every 1 and only element arranged vertically, therefore the matrix has 3 rows and 1 column.

Order of any matrices is described by the number of rows multiplied by the number of columns

Example:
State the order of the matrix below.
$\begin{pmatrix} 82 & 5 & 2 \\ 0 & 61 & 18 \end{pmatrix}$

In this case, the matrix order is said to be 2×3, as it has 2 horizontal elements for every 3 vertical elements.

Square Matrices
In the case where the matrix has same number of rows and columns, the matrix is called a square matrix.
Example:
(5) is a square matrix, as it has order 1×1
$\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$ is a square matrix, as it has order 2×2

$\begin{pmatrix} 3 & 3 & 5 \\ 0 & 4 & 1 \\ 2 & 5 & 0 \end{pmatrix}$ is also a square matrix, as it has a 3×3 order and so on….

Identity matrices are square matrices, specifically, in which all the elements in the leading diagonal are equal to 1 and the rest of the elements are equal to 0, typically denoted by the capital letter I.
Examples
$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

$\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$

Null Matrices – matrices for which all the elements inside are zero, typically denoted by 0.
Examples
$\begin{pmatrix} 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$

Addition and Subtraction Operation on Matrices
Both operation of addition and subtraction of matrices can only happen if both matrices are of the exact same order.

Example as given below

$\begin{pmatrix} 7 & 3 \\ -4 & -6 \end{pmatrix} + \begin{pmatrix} -9 & 1 \\ 6 & 7 \end{pmatrix}$ will produce valid results as the order are the same

$\begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} 8 \\ 12 \end{pmatrix}$ will not produce valid results of any kind due to differences in order of both matrices

$(5 \quad -6) - (5 \quad 7)$ will produce valid results as the order are the same

$\begin{pmatrix} 8 & 0 & 9 \\ 5 & -3 & 7 \end{pmatrix} - \begin{pmatrix} 0 & -5 \\ 2 & -1 \end{pmatrix}$ will not produce valid results of any kind due to differences in order of both matrices.

Things to note about matrices addition and subtraction
Matrices addition is commutative in nature which means, if A and B represents two matrices of the same order, then
$A + B = B + A$

Matrices addition is also associative in nature, which means if A, B and C represents three matrices of the same order, then
$(A + B) + C = A + (B + C)$

However, matrices subtraction is neither commutative nor associative in nature, as with regular subtraction, which means generally speaking,
$A - B \neq B - A$
$(A + B) - C \neq A - (B + C)$

Multiplication Operation involving Scalar and a Matrix
Multiplication Operation can involve regular numbers (also called a scalar) being multiplied to a matrix
Given a matrix, A and a regular number, n the product of the two is $n \times A$

Multiplication Operation involving Multiple Matrices
Both matrices can only be multiplied together provided, the number of columns in the first matrix matches the number of rows in the second matrix, in mathematical terms, this implies, given two matrices, A and B

Matrix A has order of $m \times n$, Matrix B has order of $p \times q$, multiplication can only occur if $n = p$

The resultant matrix AB after multiplying A and B shall have an order of $m \times q$

Demonstration of how to perform addition and subtraction of matrices, along with scalar multiplication
Example (Performing scalar multiplication to a matrix directly)
In the below case, the number 3 is to be multiplied by every single element in the bracket, which implies

$$3\begin{pmatrix} 8 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \times 8 \\ 3 \times (-2) \end{pmatrix} = \begin{pmatrix} 24 \\ -6 \end{pmatrix}$$

Example (Performing Addition and Subtraction of Matrices Only)

Given matrix $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ and $B = \begin{pmatrix} f & g \\ h & k \end{pmatrix}$

$$A + B = \begin{pmatrix} a+f & c+g \\ b+h & d+k \end{pmatrix}$$

$$\begin{pmatrix} 5 & 8 \\ 1 & 8 \end{pmatrix} + \begin{pmatrix} 6 & 2 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 11 & 10 \\ 8 & 16 \end{pmatrix}$$

Given matrix $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$ and $B = \begin{pmatrix} f & g \\ h & k \end{pmatrix}$

$$A - B = \begin{pmatrix} a-f & c-h \\ b-h & d-k \end{pmatrix}$$

$$\begin{pmatrix} 5 & 8 \\ 1 & 8 \end{pmatrix} - \begin{pmatrix} 6 & 2 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ -6 & 0 \end{pmatrix}$$

Examples (Performing Addition and Subtraction of Matrices, alongside with scalar multiplication)

$$3\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} - 2\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \\ 12 \end{pmatrix} - \begin{pmatrix} 10 \\ 12 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ -21 \\ 12 \end{pmatrix}$$

$$3\begin{pmatrix} -2 & 1 \\ 0 & 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 8 & -6 \\ 10 & 12 \end{pmatrix} = \begin{pmatrix} -6 & 3 \\ 0 & 12 \end{pmatrix} + \begin{pmatrix} 4 & -3 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 5 & 18 \end{pmatrix}$$

Demonstration of how to multiply a matrix by another matrix
Step 1. Read the question carefully and note the order of both matrices
Step 2. Determine the order of the resultant matrix after both are multiplied together
Step 3. Multiply the matrices together according to the guideline below

$$\begin{pmatrix} 1 \\ 6 \end{pmatrix}\begin{pmatrix} 2 & 3 \end{pmatrix}$$

In the above example, we noted that the matrix order of both matrices are 2×1 and 1×2 respectively, so the resultant matrix must have an order of 2×2

Every time when a matrix is multiplied by another matrix, it is always a matter of multiplying the rows of a matrix by the columns of another matrix and adding the product together in the following manner, as seen in the following few examples.

Examples (Performing multiplication of matrices together)

$$\begin{pmatrix} 1 \\ 6 \end{pmatrix}\begin{pmatrix} 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 \times 2 & 1 \times 3 \\ 6 \times 2 & 6 \times 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 12 & 18 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & 3 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \times (-1) + 2 \times 2 + 5 \times 3 \\ 0 \times (-1) + 3 \times 2 + (-1) \times 3 \end{pmatrix} = \begin{pmatrix} 18 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix}1 & 2\\0 & -1\end{pmatrix}^2=\begin{pmatrix}1 & 2\\0 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\0 & -1\end{pmatrix}=\begin{pmatrix}1\times1+2\times0 & 1\times2+2\times(-1)\\0\times1+(-1)\times0 & 0\times2+(-1)\times(-1)\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

Applications of Matrices

Andy and Mark each purchased some pencils, pens and erasers. The table below show the number of stationary purchased and the cost of each item.

	Pencil	Pen	Eraser
Andy	5	6	2
Mark	8	4	0

	Cost
Pencil	0.20
Pen	1.50
Eraser	0.40

(a) Write two matrices that can represent the above information and multiply the matrices together, taking into account the order provided by the table.

(b) Explain what your answer to part (a) represents

(a)

In this case, we get the following matrices, which are multiplied together in the following manner.

[As the matrix on the left is a 2×3 matrix and the matrix on the right is a 3×1 matrix, we will get a 2×1 as a product of the 2]

$$\begin{pmatrix}5 & 6 & 2\\8 & 4 & 0\end{pmatrix}\times\begin{pmatrix}0.20\\1.50\\0.40\end{pmatrix}=\begin{pmatrix}5\times(0.20)+6\times(1.50)+2\times(0.40)\\8\times(0.20)+4\times(1.50)+0\times(0.40)\end{pmatrix}=\begin{pmatrix}10.80\\7.60\end{pmatrix}$$

(b)

My answer to part (a) represents the amount of money Andy and Mark spent on stationary respectively, where Andy spent \$10.80 and Mark spent \$7.60.

Title	Principles of Money Exchange
Author	Liu Hui Ling, Student of Ngee Ann Polytechnic
Date	29/5/2018

Before we start introducing this topic, let us talk about why different country has different currencies in the first place. There isn't a short answer for that but these are the following reasons:

- Origins of Money – The origin of money is different for countries, when they switch over to notes, their origin is taken into account as well.
- Politics – As a political leader, you want some control over your country's economic system and thus, would use your own country's currency.

As the origin of money is different in different countries, we cannot just say, let the world adopt a common currency and call it a day, this whole story is further extended by political reasons where leaders want control over their nation's spending.

Notation in this topic	Examples and Meaning
Currency Notation	Usually a 3 Letter (Sometimes 2 or 4, but rare) joined together, that either abbreviates the currency name or just the currency prefix. USD – United States Dollars JPY – Japanese Yen EUR – Euros
Amount of Money in that Currency	Amount in the currency, followed by the 3-Letter-Representation of that currency. 200 EUR – Means 200 Euros 115 JPY – Means 200 Japanese Yen Occasionally symbols are used as well but I rarely see in textbooks, in the context of currency exchange. However, if you wish to go for overseas trip, do a good research on their currency symbols, as I mentioned, the origins of money are different in various countries, so could be the language and thus they also adopt different symbols for their native currency.

Just some extra knowledge I think would be useful if you are planning for overseas learning trips, these terms may be used frequently in the process of exchanging currency with a money changer, or perhaps when you study Principle of Accounting next time.

Extended Terms	Meaning
Appreciation	To rise in values in relation to other currency, comparing with the past
Depreciation	To decrease in value in relation to other currency, comparing with the past.

Example question (Disclaimer: This is just an example and may not be true.)

Given the below currency rates, solve the following questions.

1 USD	0.78 SGD
1 SGD	0.64 EUR

Alice wishes to buy a present for her friend as a gift on birthday from an online store. According to the site, any items sold on that site have their values all calculated in Euros. Given that the present is 19 Euro, and that she needs to also pay an additional 11 Euro in transportation fees and 7 SGD worth of import charges. How much does she as to pay at the end in Singapore Dollars?

Step 1. Calculate currency rates from table information.
Since 1 SGD is 0.64 EUR, we calculate what is 1 EUR in SGD.

1 SGD = 0.64 EUR

$$\frac{1\ SGD}{0.64} = \frac{0.64\ EUR}{0.64}$$

1.5625 SGD = 1 EUR

(For easy calculation, switch the target currency to the Right-Hand Side)
1 EUR = 1.5625 SGD

Present + Transportation Fee (in Euro) =
19 EUR + 11 EUR = 30 EUR

Convert to target currency, (in Singapore Dollars)
30 EUR = 30(1.5625) SGD = 46.875 SGD

Add Charges (in Singapore Dollars)
46.875 SGD + 7 SGD = 53.875 SGD

Total = 53.875 SGD
Total (Currency Round off to 2 Decimal Places): 53.88 SGD

Bob's manager has a task for Bob. He is asked to travel to the United States to train the staff there on using accounting software. His company sponsored 6000 SGD of trip fees for his accommodation in United States during his trip.

Given he spent 1500 USD in the United States, how much money is he left with in SGD after the trip ends?

Convert all values to target currency
1 SGD = 0.78 USD
1 USD = 1.28205 SGD (I use 5 decimal places for this calculation, you can use fraction as well.) (Money changers, typically, don't use scientific calculators for calculation and thus unable to use fraction values to calculate.)

Convert Expenses to Target Currency Value
1500 USD = 1500(1.28205) = 1923.075 SGD

Money Left (in SGD) = 6000 SGD – 1923.075 SGD = 4076.925 SGD,
4076.93 SGD (2 Decimal Places)

Title	Mathematics in Practical Situations – Simple and Compound Interest – Basic Explanation and Examples
Author	Liu Hui Ling, Ngee Ann Polytechnic
Date	19/7/2018

Objectives of this lesson is to allow students to

- Understand the idea of simple and compound interest
- Understand the formula for simple and compound interest

I am studying business related modules and know what students will face, particularly those not so mathematically inclined students. I am here to help explain the concepts in an easy-to-understand manner.

Simple Interest Properties

- Amount of Interest Obtained per time interval is <u>constant</u>
- Amount of Interest Obtained per time interval is calculated by multiplying the interest rate by the original amount put in to a system or given (Can be bank loan, or bank deposit)

Simple Interest (Mathematical Details)

For interest value calculation
$I = PRT$

I refers to the interest amount obtained
P stands for principle amount, refers to the amount of money put into the system or loaned
R refers to the interest rate in %
T number of time intervals passed after the money is put into a system

For amount of money in the system or to be returned after T number of time intervals is
$A = P(1 + RT)$
A refers to the total amount after T number of time intervals.

Explanation of the Simple Interest Formula as Given Below
Imagine you are a customer for a bank and you put in $2300 in your bank account. Based on the agreement, the bank was to offer you an interest rate of 3% per year.

Number of Years Passed T	Interest Amount I	Final Amount A
1	$\$2300 \times 3\% \times 1$	$2300 + (2300 \times 3\% \times 1)$
2	$\$2300 \times 3\% \times 2$	$2300 + (2300 \times 3\% \times 2)$
3	$\$2300 \times 3\% \times 3$	$2300 + (2300 \times 3\% \times 3)$

The final amount as you can see from the pattern can be expressed as the following formula

$A = P + (PRT)$
$A = 1(P) + RT(P)$

Right Hand Side to be factorized as follows.
$A = P(1 + RT)$

Compound Interest Properties

- Interest Amount keep increasing with Each Passing Time Interval
- Interest Rate is constant however, the interest amount is calculated from the amount from the previous time interval. (I will explain in detail below)

Imagine you went into a bank and deposited $12000 into a new bank account. The bank is to allow you to earn 4% of interest to be compounded yearly.

Number of Years Passed T	Interest Amount I	Final Amount After T number of years passed A
1	12000(4%) =	$12000(4\%) + 12000 = 480 + 12000 = \$12480$ Or $12000 \times 104\% = \$12480$
2	12000(4%) + 12480(4%) = 480+499.2 = $979.20	$12000\times 104\% \times 104\% = \$12979.20$

3	12000(4%) + 12480(4%) + 12979.20(4%) = 480+499.2+519.168 = $1498.368	$12000 \times 104\% \times 104\% \times 104\% =$ 13498.368

As you can see from the above, a pattern emerges.

A is the final amount obtained after T number of time intervals.
r is the amount of percentage interest given, $\frac{r}{100}$ refers to the percentage interest itself.
We derived the following formula for amount of money obtained, after T number of time intervals and $r\%$ of compound interest.

$$A = P\left(1 + \frac{r}{100}\right)^{T}$$

(Some textbooks may use different letters. Just change the letters accordingly and you should get the idea.)

Title	Financial Arithmetic – Hire Purchase
Author	Liu Hui Ling, Ngee Ann Polytechnic
Date	7/1/2019

Hire purchase, otherwise known more familiar as "pay by installments" appears quite often in our lives. That may include purchasing a vehicle or any assets for which paying at one go isn't always a feasible idea, therefore there will be a need to divide the payment into equal segments or "installments" and often it will take many years before you finally clear the installments.

Terminology used in Hire Purchase plans

- Deposit/Initial Payment – The amount of money being paid upfront at the point of purchase.
- Installments – The number of "intervals" for which the payment is divided into equal segments.

Structure of a Hire Purchase Plan Which Are Important To Note

Initial payment amount	Example: A car cost $\$50\ 000$ and the buyer pay an initial amount of 40%, implying the initial amount is $\$50\ 000(40\%) = \$20\ 000$
Number of Installments	Example: The car dealer allows you to pay the remaining amount in 24 months, each payment to be done every month, which works out to 24 installments in total.
Interest Rate (Typically simple interest)	The car dealer also requires you to pay an additional 2.5% interest on top of your total installment amount.
Total Interest payable	Example: The total interest can be computed as follows: Total Interest payable = Remaining Amount× Interest Rate In the car example, the remaining amount is $\$50\ 000 - \$20\ 000 = \$30\ 000$ Total interest payable is $\$30\ 000\ \times 2.5\% = \$750$

Amount Paid per Installment	Amount Paid per Installment = $\frac{Remaing\ Amount + Total\ Interest\ Payable}{Number\ of\ Installment}$ Using the car example, we can calculate the Amount paid per installment as $$\frac{\$30\ 000 + \$750}{24} = \$1281.25$$
Total Amount Payable	Total Amount Payable is computed as follows $Total\ Amount\ Payable = (Initial\ Payment\ Amount + Remaining\ Amount + Total\ Interest\ Payable)$ Which works out to be $\$20\ 000 + \$30\ 000 + \$750 = \$50\ 750$